

Exmouth Community College

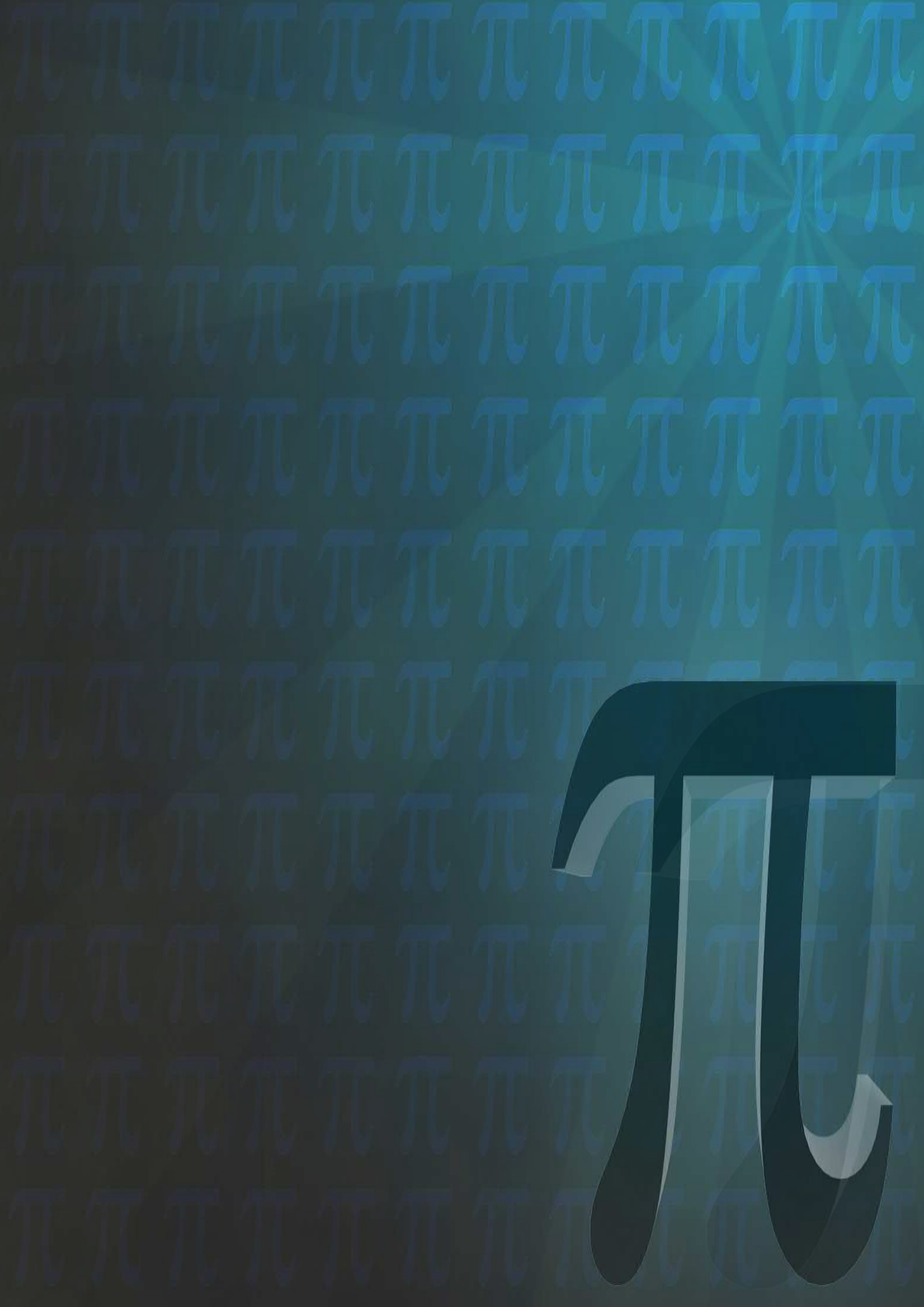
KS4 Knowledge Organisers

MATHEMATICS

Units 1 - 10

FOUNDATION





## Unit 1 Foundation Number

**BIDMAS** is the acronym to give the priority of operations:

**Brackets, Indices** (powers and roots),  
**Division AND Multiplication, Addition AND Subtraction**

Do anything in brackets first, then any indices, then, from left to right, and divisions or multiplications, then, from left to right, any additions or subtractions.

[Video 211 - https://tinyurl.com/y98jn4wk](https://tinyurl.com/y98jn4wk)

= means equals

≠ means not equals

≈ means roughly equals

A **function** is a rule that acts on a number.  
Eg)  $x2$  (times 2)

An **inverse function** reverses the effect of a function

+ and - are inverse operations

$\times$  and  $\div$  are inverse operations

Key Points:



<https://tinyurl.com/y7zu7719>

Squaring a number means multiplying it by itself. The result is a **square number**, e.g.  $4^2 = 4 \times 4 = 16$  which is a square number

[Video 226 - https://tinyurl.com/ya4v48rn](https://tinyurl.com/ya4v48rn)

Cubing a number means multiplying it by itself twice. The result is a **cube number**, e.g.

$4^3 = 4 \times 4 \times 4 = 64$  which is a cube number

[Video 212 - https://tinyurl.com/ydd72o3d](https://tinyurl.com/ydd72o3d)

The **square root** of a number is the number you must square to get the original number. It is the inverse of squaring  $\sqrt{16} = 4$

[Video 228 - https://tinyurl.com/yc28q7lv](https://tinyurl.com/yc28q7lv)

The **cube root** of a number is the number you must cube to get the original number. It is the inverse of cubing, e.g.  $\sqrt[3]{64} = 4$

[Video 214 - https://tinyurl.com/y9q9m7nb](https://tinyurl.com/y9q9m7nb)

A prime number has two factors, itself and 1, - e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23...

[Video 225 - https://tinyurl.com/ybnk7z5n](https://tinyurl.com/ybnk7z5n)

To multiply powers of the same number, add the indices, e.g.  $4^3 \times 4^8 = 4^{11}$

To divide powers of the same number, subtract the indices, e.g.  $4^8 \div 4^3 = 4^5$

[Video 174 - https://tinyurl.com/za9u7h2](https://tinyurl.com/za9u7h2)

Knowledge Check:



<https://tinyurl.com/ya7obwjs>

Rounding is where you approximate a number to make it more manageable.

If we round to decimal places, we get rid of all digits after the required decimal place. The final decimal place goes up by one if the first digit we ignore is 5 or more, e.g.  $4.597 = 4.6$  (1 d.p.)

[Video 278 - https://tinyurl.com/y9x7ltoh](https://tinyurl.com/y9x7ltoh)

If we round to significant figures, we get rid of all digits after the required digits from the left (ignoring leading zeros). The final digit goes up by one if the first digit we ignore is 5 or more, e.g.  $0.0465 = 0.047$  (2 s.f.)

[Video 279a - https://tinyurl.com/yakhqfup](https://tinyurl.com/yakhqfup)

To estimate we round all numbers in a calculation to 1 significant figure (1 s.f.).

A **factor** is a number you can multiply by to get a desired number, e.g. 2 is a factor of 8  
[Video 117 - https://tinyurl.com/zymmfev](https://tinyurl.com/zymmfev)

A **multiple** is a number you can divide by an integer to get a desired number. Eg) 16 is a multiple of 8

[Video 220 - https://tinyurl.com/yaudfco3](https://tinyurl.com/yaudfco3)

**Highest Common Factor (HCF)** is the highest factor that is common to two or more numbers, e.g. 4 is the HCF of 8 and 12

[Video 219 - https://tinyurl.com/zell3pzq](https://tinyurl.com/zell3pzq)

**Lowest Common Multiple (LCM)** is the lowest multiple that is common to two or more numbers, e.g. 24 is the LCM of 8 and 12

[Video 218 - https://tinyurl.com/y8hg8z35](https://tinyurl.com/y8hg8z35)

# Unit 2 Foundation Algebra

A term is a number, a letter, or a number and a letter multiplied together, e.g. 3, a, 2b, 4c<sup>2</sup>  
[Video 19 - https://tinyurl.com/hgw9ulw](https://tinyurl.com/hgw9ulw)

Letters represent variables; the value can vary. Like terms contain the same letters or power of letters, or are just numbers, e.g. 3 and 4, 3a and 6a, b<sup>3</sup> and 2b<sup>3</sup>

To simplify an expression we can collect like terms, e.g. 3a + 2 + 4a = 7a + 2  
[Video 9 - https://tinyurl.com/z77lutd](https://tinyurl.com/z77lutd)

We can also simplify multiplications by removing the multiplication symbol and divisions by making into a fraction, e.g. 2 x a = 2a, c ÷ d = c/d or  $\frac{c}{d}$

If we have an expression or equation and are given the value of a variable, we can substitute this value in, e.g. 3a + b = c where a = 2 becomes 6 + b = c

[Video 20 - https://tinyurl.com/zd6tv9j](https://tinyurl.com/zd6tv9j)

**Key Points:**



<https://tinyurl.com/y9j5u8ws>

A **formula** shows the relationship between terms, e.g. 4a + b = c

An **expression** is a collection of terms, e.g. 2a + 1

An **equation** is an expression equalling another, e.g. 3b + 2 = 2d

An **inequality** is where two expressions don't, or don't necessarily, equal each other (<, >, ≤, ≥), e.g. 4f > 6

An **identity** is two expressions that always equal each other, regardless of the variables, e.g. 2(a + 5) ≡ 2a + 10

A **not equal** symbol shows that two expressions do not equal each other, e.g. 2a ≠ b

[Video 16 - https://tinyurl.com/j5cdu68](https://tinyurl.com/j5cdu68)

To multiply terms, multiply any numbers, put non-like terms next to each other, and add powers of like terms, e.g. 2a x 3a x 4b = 24a<sup>2</sup>b

[Video 18 - https://tinyurl.com/ybaxlv6k](https://tinyurl.com/ybaxlv6k)

To multiply the same variable with powers, add the indices, e.g. 2a<sup>2</sup> x 4a<sup>3</sup> = 8a<sup>5</sup>

To divide the same variable with powers, subtract the indices, e.g. 8a<sup>5</sup> ÷ 2a<sup>3</sup> = 4a<sup>2</sup>

[Video 11 - https://tinyurl.com/ycvjt5b](https://tinyurl.com/ycvjt5b)

**Knowledge Check:**



<https://tinyurl.com/yb8a3eto>

To **expand brackets**, multiply the terms in the brackets by the multiplier, e.g. 5(a + 2) = 5 x a + 5 x 2 = 5a + 10

[Video 13 - https://tinyurl.com/hepjutn](https://tinyurl.com/hepjutn)

To expand **double brackets**, multiply every term in one bracket by every term in the other, e.g. (a + b)(c + d) = a x c + a x d + b x c + b x d = ac + ad + bc + bd

[Video 14 - https://tinyurl.com/ycptvous](https://tinyurl.com/ycptvous)

To **factorise** expressions we reverse the expansion of brackets. We do this by dividing through by the **HCF** (highest common factor) and putting the HCF as the multiplier outside the brackets, e.g. 5a + 10b = 5(a + 2b)

[Video 117 - https://tinyurl.com/zymmfev](https://tinyurl.com/zymmfev)

To rearrange an equation (or inequality), always do the same to both sides of the equation and use the opposite operator to remove a term, e.g. a + 2b = c [- a]

$$2b = c - a \quad [ \div 2 ]$$

$$b = \frac{c - a}{2}$$

We use this to change the subject of a formula.

[Video 110 - https://tinyurl.com/y866296z](https://tinyurl.com/y866296z)

# Frequency Tables

These are a useful and clear way of displaying data, e.g. the table below shows the scores out of ten for 20 students.

Mark	Tally	Frequency
4		2
5		2
6		4
7		5
8		4
9		2
10		1

Frequency means how often something occurs.

This means 5 students scored 7 marks in their test.

# Graphs Tables and Charts (Unit 3 Foundation)

## Two-Way Tables

[Video 319](#)

These are used to show how data falls into 2 different categories, for example gender and favourite sport to watch.

**What is your favorite sport to watch on television?**

	Football	Basketball	Baseball
Males	40	22	15
Females	12	16	45
Total	52	38	60

A two-way table divides data into groups in rows going across and columns going down the table.

[Video 169](#)

## Stem and Leaf Diagrams

[Video 170](#)

This shows numerical data split into a 'stem' and 'leaves'. The leaf is usually the last digit and the stem is the other digits.

Here are the heights of some students (in cm).  
169, 163, 153, 173, 166, 178, 177  
Construct a stem and leaf diagram for this data.

15 | 3  
16 | 9 3 6  
17 | 3 8 7

Decide on a stem. Write the numbers in your diagram as you work along the data list.

15 | 3  
16 | 3 6 9  
17 | 3 7 8

Put the leaves in your diagram in order.

Key: 15|3 means 153 cm

Write a key for your diagram.

A back-to-back stem and leaf diagram compares two sets of data, e.g. the ages of males and females.

Females	Males
7 5 4 3 0 8	6 7 9
9 8 6 1 9	8 3 5
	9 2 3 5 7 8
	10 1 3 7

5 | 7 means 75

10 | 1 means 101

## Grouped Frequency Tables

These contain sorted data in groups called **classes**, e.g. the table below shows the ages of people taking swimming lessons.

Class Interval	Frequency
15 – 25	60
25 – 35	35
35 – 45	22
45 – 55	18
55 – 65	15

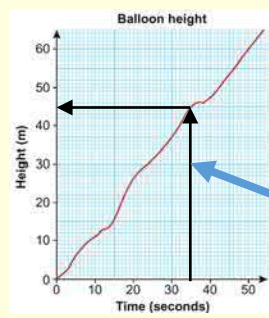
Total frequency will tell you the total number of people taking swimming lessons.

This means 18 people who took swimming lessons were between the ages of 45 and 55.

Classes or class widths

## Time-Series Graph

These are used to show how something changes over time. It is a line graph with time plotted along the horizontal axis. For example the height of a balloon at different times



You can estimate the height of the balloon at different times using the graph

E.g. the height of the balloon at 35 seconds is approximately 45m as shown by the arrows on the graph

[Video 163 - Drawing](#)

## Pie Charts

[Video 164 - Interpreting](#)

This is a circle divided into **sectors**. Each sector represents a set of data. Pie charts are excellent for displaying the most/ least popular type of something.

### Plotting pie charts example

The table show the match results of a football team.

Result	Won	Drawn	Lost
Frequency	28	12	20

$28 + 12 + 20 = 60$

The total number of games is the total frequency.

1 game =  $360^\circ \div 60 \text{ games} = 6^\circ \text{ per game}$

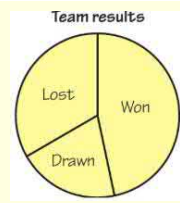
Work out the angle for one game.

28 games won =  $28 \times 6^\circ = 168^\circ$

12 games drawn =  $12 \times 6^\circ = 72^\circ$

20 games lost =  $20 \times 6^\circ = 120^\circ$

Work out the angle for each result.



Draw the pie chart. Give it a title and a key. Or label each section

## Comparative Bar Charts

The table shows the number of cars sold by Kitty and George in the first four months of 2014.

[Video 147](#)  
[Video 148](#)

	January	February	March	April
Kitty	2	5	13	10
George	4	7	9	10



The chart has a key to make it easier to understand.

A comparative bar chart allows you to easily compare the number of cars Kitty and George sold each month.

## Scatter Graphs

A scatter graph allows you to see the **relationship** between two sets of data, e.g. your height and your stride length. Correlation is used to describe a relationship between two **variables**.

[Videos 165 - 168](#)

**Positive correlation**

**Negative correlation**

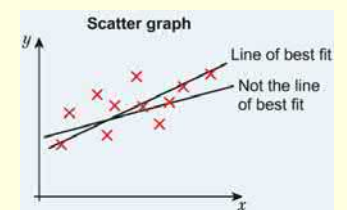
**No correlation**

**Outlier**

An outlier is a value that does not fit the pattern of data.

### A line of best fit


This is a straight line drawn through the middle of the points on a scatter graph. It should pass as near as many points as possible and represents the **trend** of the points.



A line of best fit can be used to predict data values within the range of data given. This is called **interpolation**. It can also be used to predict data values outside the range of data given. This is called **extrapolation**.


# Fractions

**The basics:**  
This pizza is  $\frac{3}{4}$  shaded green



3 is the "numerator"  
4 is the "denominator"

Notice that  $\frac{6}{8}$  is exactly the same amount. (both numbers doubled)



**Multiplying fractions:**  
Just multiply numerators, multiply denominators, and **simplify** if possible

$$\frac{2}{4} \times \frac{2}{4} = \frac{4}{16} = \frac{1}{4}$$

Simplifying involves dividing numerator and denominator by their HCF  
...HCF is the Highest Common Factor

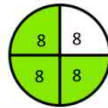
**Fractions of amounts:**

Use simpler fractions to find the fraction you actually want:

Eg.  $\frac{1}{4}$  of 32:  $\frac{1}{4}$  of 32 =  $32 \div 4 = 8$   
so  $\frac{3}{4}$  of 32 =  $8 \times 3 = 24$

Divide by the denominator, then multiply by the numerator

In this example, a whole pizza = 32



## Simplifying fractions:

Divide numerator and denominator by HCF. You should do this to any final answer fraction where possible.

# Percentages of amounts

**Calculator allowed?**

Turn % into decimal ( $\div 100$ ) and "of" means "multiply".

e.g. 30% of £54 =  $30 \div 100 \times 54 = £16.20$

e.g. 28% of £40 =  $28 \div 100 \times 40 = £11.20$



**Calculator not allowed?**

10% is your starting point. 10% means "a tenth of the amount" (because  $10\% = 10/100 = 1/10$ )



You can work out all the other percentages you need by scaling up or down from 10%

e.g. 30% of £54?

10% = £5.40 (a tenth of 54 = 54/10)  
20% = £10.80 (20% is double 10%)  
30% = £16.20 (30% = 10% + 20%)

e.g. 28% of £40?

10% = £4

1% = 40p (divide 10% by 10)  
2% = 80p (double 1%)  
5% = £2 (half 10%)  
20% = £8 (double 10%)

28% = these 4 added together, = £11.20

**Reverse percentages:**

Use the logic of function machines, which can be run backwards.

You need to figure out the forwards multiplier first.

e.g. \$30 dress reduced by 20%:

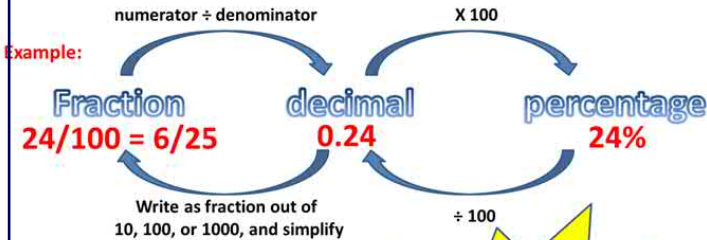
$$\$30 \times 0.8 = \$24$$

e.g. Sale price after 30% discount = £28

$$\begin{matrix} \text{Original price} & ? & \times 0.7 & \text{£28} \\ \text{£40} & \div 0.8 & & \text{£28} \end{matrix}$$



# Fractions, decimals, percentages conversion



**Some examples:**

- $\frac{1}{10} = \frac{10}{100} = 0.1 = 10\%$
- $\frac{1}{5} = \frac{20}{100} = 0.2 = 20\%$
- $\frac{3}{10} = \frac{30}{100} = 0.3 = 30\%$
- $\frac{9}{20} = \frac{45}{100} = 0.45 = 45\%$

People often assume a % cannot be over 100, but it can (just like a fraction can be improper\* and a decimal can be over 1)

\* top-heavy

## Fractions:

To multiply fractions, just multiply numerators and denominators:

e.g.  $\frac{2}{7} \times \frac{4}{5} = \frac{8}{35}$

To divide fractions, KFC (keep, flip, change)

e.g.  $\frac{2}{7} \div \frac{4}{5} = \frac{2}{7} \times \frac{5}{4} = \frac{10}{28}$

## Battenburg: adding

1. Draw the battenburg grid.
2. Put the fractions on the side, (left to right, top to bottom).
3. Eat the top left corner (cross it out).
4. Do the multiplications.
5. "ADD the peanut" (the yellow ones below).
6. Peanut answer is numerator, the remaining number is denominator.
7. Simplify the fraction, if possible.

$$\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$

	1	4
1	X	4
3	3	12

Divide top and bottom of fraction with the HCF that they share

## Battenburg: subtracting

1. Draw the battenburg grid.
2. Put the fractions on the side, (left to right, top to bottom).
3. Eat the top left corner (cross it out).
4. Do the multiplications.
5. "SUBTRACT the peanut" (the yellow ones below).
6. Peanut answer is numerator, the remaining number is denominator.
7. Simplify the fraction, if possible.

$$\frac{1}{4} - \frac{1}{3} = \frac{1}{12}$$

	1	4
1	X	4
3	3	12

Divide top and bottom of fraction with the HCF that they share

# Equations, Inequalities, Sequences (Unit 5 Foundation)

An **equation** contains an unknown number (letter) and an equals (=) sign.

You **solve** an equation by working out the value of the unknown.

[Video 110 - https://tinyurl.com/y866296z](https://tinyurl.com/y866296z)

In an equation, both sides of the = sign have the same value (like balanced scales). As with balanced scales, the two sides remain equal if the same is done to both sides (**balancing method**).

In an equation with **brackets**, expand the brackets first.

To expand brackets, multiply everything within the brackets by any multiplier on the outside.

A **formula** is an equation with two or more **variables** (unknown numbers).

Values can be **substituted** into a formula to get results.

[Video 113 - https://tinyurl.com/y76yatx2](https://tinyurl.com/y76yatx2)

Key Points:



<https://tinyurl.com/y9cavj7r>

An **integer** is a positive or negative whole number, or a zero.

< means **less than** (the thing on the left is less than the thing on the right)

> means **greater than** (left side greater than right side)

≤ means **less than or equal to** (like less than, but the two sides might be equal)

≥ means **greater than or equal to** (like greater than but the two sides might be equal)

[Video 176 - https://tinyurl.com/y7py6cf9](https://tinyurl.com/y7py6cf9)

You **MUST** do the **SAME** to **BOTH** sides of an equation or inequality

[Video 178 - https://tinyurl.com/hkxkrvk](https://tinyurl.com/hkxkrvk)

**Inequalities** can be shown on number lines with empty circles (for less than or greater than) or filled circles (if value could be equal) and arrows in correct direction.

[Video 177 - https://tinyurl.com/y72g4v69](https://tinyurl.com/y72g4v69)

Knowledge Check:



<https://tinyurl.com/y96fhs9v>

**Sequences** are patterns of numbers that follow a rule.

The numbers in a sequence are called **terms**.

[Video 286 - https://tinyurl.com/ydaj355k](https://tinyurl.com/ydaj355k)

The **term-to-term** rule describes how to get from one term to the next.

[Video 287 - https://tinyurl.com/y7mp8hdf](https://tinyurl.com/y7mp8hdf)

The **nth** term of a sequence is how to work out the term given its position ( $n$ ) in the sequence.

[Video 288 - https://tinyurl.com/hs9ansx](https://tinyurl.com/hs9ansx)

The **nth** term is sometimes called the **general term** of a sequence.

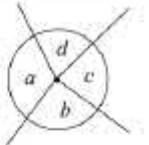
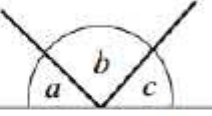
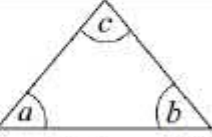
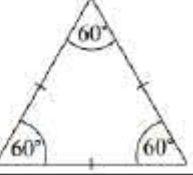
In a **linear sequence** (same difference between each pair of terms) the  $n$ th term is found by multiplying the position by the difference between the first and second terms, then adding or subtracting a constant to make the output when  $n = 1$  actually equal the first term.

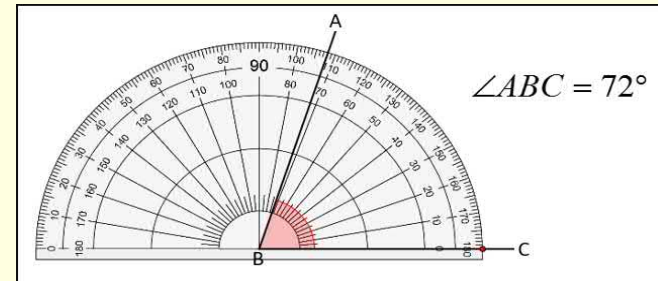
As with all mathematical calculations, please remember to use **BIDMAS**:

**Brackets** then **Indices** then **Division & Multiplication** then **Addition & Subtraction**

[Video 211 - https://tinyurl.com/y98jn4wk](https://tinyurl.com/y98jn4wk)

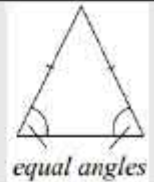
# ANGLES (Unit 6 Foundation)

Angles at a point add up to 360°.	 $a + b + c + d = 360^\circ$
Angles on a straight line add up to 180°.	 $a + b + c = 180^\circ$
The interior angles in any triangle add up to 180°.	 $a + b + c = 180^\circ$
The interior angles in an equilateral triangle are all 60°.	

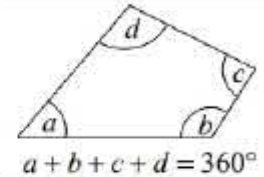


Videos	
Names of angles	<a href="#">V38</a>
Angles in a triangle	<a href="#">v37</a>
Angles on a line/ around a point	<a href="#">V35</a> <a href="#">V30</a>
Angles and parallel lines	<a href="#">V25</a>
Properties of special triangles	<a href="#">V327</a>

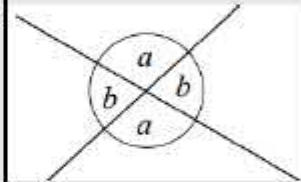
An isosceles triangle has two angles of the same size.



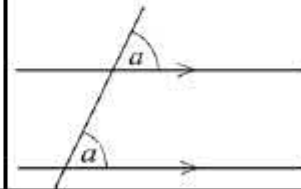
The interior angles in any quadrilateral add up to 360°.



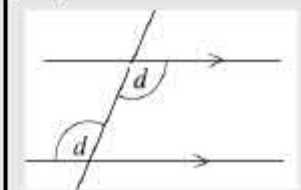
When two straight lines intersect, the opposite angles are equal.



When a straight line intersects a pair of parallel lines, the **corresponding angles** are equal.

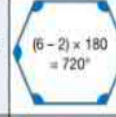




When a straight line intersects a pair of parallel lines, the **alternate angles** are equal.



Angle	Vertically opposite
Alternate	Perpendicular
Supplementary Co-interior	Parallel lines
Acute/Obtuse/ Reflex	corresponding

## Key facts to memorise- polygon angle facts

Polygon names		Polygon angle facts	
3 sides	Triangle	Sum of interior angles in a polygon with n sides = $(n - 2) \times 180$	
4 sides	Quadrilateral		
5 sides	Pentagon		
6 sides	Hexagon	Sum of exterior angles in a polygon = 360°	
7 sides	Heptagon		
8 sides	Octagon	Interior angle + exterior angle = 180°	
9 sides	Nonagon		
10 sides	Decagon		



# Averages and Range (Unit 7 Foundation)

## MEAN

$$\frac{\text{Sum of all values}}{\text{Number of values}}$$

## MEDIAN

Middle value when numbers are placed in order

## MODE

Most Common

## RANGE

Largest value – smallest value

### Averages and Range from a Frequency table

20 students scored goals for the school hockey team last month. The table gives information about the number of goals they scored.

Goals scored	Number of students	Goals scored x no. of students
1	9	1 x 9 = 9
2	3	2 x 3 = 6
3	5	3 x 5 = 15
4	3	4 x 3 = 12
20 students		42 goals scored

This means 9 students each scored 1 goal

Add a totals row to work out the total number of students and total goals scored

Add an extra column to work out the number of goals scored

**Mean** = Total number of goals scored divided by the number of students  
 =  $42 \div 20 = 2.1$  goals per student

**Mode** = Most common number of goals scored  
 = 1 (as 9 students scored 1 goal which is more than any other number of goals)

**Median** = The number in the middle = 2  
 If I wrote the goals scored by each student as a list it would look like:

1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4

9 students scored 1 goal each      3 students scored 4 goals each

The **median** is the middle number now that they are in order.

**Range** of the number of goals scored =  $4 - 1 = 3$

This stem and leaf diagram shows the times, in seconds, for a group of swimmers to swim 100m. Find the median and the mode.

55	2	3	6			
56	3	3	7	8		
57	0	2	6	6	6	7
58	4	4	5			
59	3					

Count the number of values; 17.  
 The median is the  $\frac{n+1}{2}$ th value. There are 17 values, so  $n = 17$ .  
 $\frac{17+1}{2} = 9$   
 The median is the 9th value. The median is 57.2 seconds.  
 The mode is 57.6 seconds.

Key: 55|2 means 55.2 seconds

In a stem and leaf diagram the data is in order. So count up to the 9th value.  
 Look for repeated values in the rows.  
 57| 0 2 6 6 6 7

### Estimating the mean from a grouped frequency table

[Estimated mean video](#)

The table represents the scores of 30 students in a Maths test:

Score	Frequency
1-5	5
6-10	6
11-15	9
16-20	10

Work out an estimate for the mean.

Score	Frequency, $f$	Midpoint of class, $m$	$m \times f$
1-5	5	3	15
6-10	6	8	48
11-15	9	13	117
16-20	10	18	180
<b>Total</b>	<b>30</b>	<b>Total</b>	<b>360</b>

Estimate of mean =  $\frac{360}{30} = 12$

Add a column to calculate the midpoint of each class. Use this as an estimate of the scores, because you don't know the exact values in each class.  
 Add a column,  $m \times f$ , to calculate an estimate of the total score for each class.  
 Divide the total of the  $m \times f$  column by the total frequency.

Our data has been grouped into classes

This is only an **ESTIMATE** for the mean as we are estimating the scores of the students by using the mid-point.

### Practice Question:

**Real** In a survey, 30 small companies were asked how many employees they had. This table shows the results.

Number of employees	Frequency, $f$	Midpoint of class, $m$	$m \times f$
1-5	12		
6-10	7		
11-15	6		
16-20	5		
<b>Total</b>		<b>Total</b>	

Calculate an estimate for the mean number of employees per company.

**Practice:**  
 Find the mean, median, mode and range of the following set of numbers

1, 3, 2, 8, 7, 9, 5, 4, 10, 2, 4,

**Challenge Question**

12	6	15	?
----	---	----	---

The mean of these 4 cards is 10, what is the missing number?

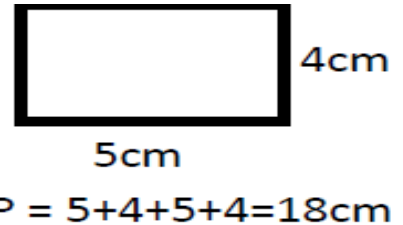
# Perimeter, Area and Volume

## (Unit 8 Foundation)

VIDEOS: [V44](#) [V45](#) [V49](#) [V40](#) [V355](#)

### Perimeter

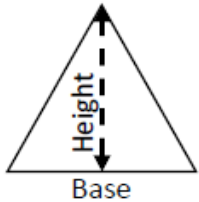
Is the distance round the edges of the shape



### Area

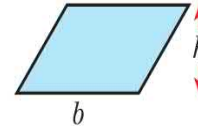
Is the inside of a shape.

Area of **Rectangle** = length  $\times$  width

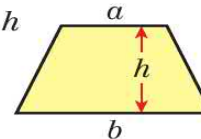


Area of **Triangle** =  $\frac{1}{2} \times \text{base} \times \text{height}$

Area of a parallelogram = base  $\times$  vertical height  
 $= b \times h$

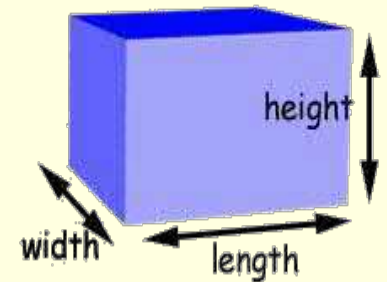


Area of trapezium =  $\frac{1}{2} \times (a + b) \times h$



### Volume

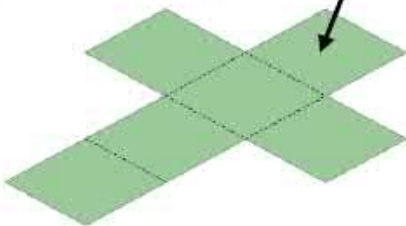
volume = length  $\times$  width  $\times$  height



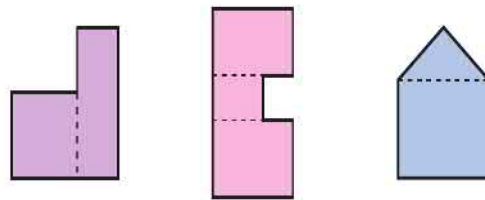
### Surface

The area of each face added together.

Face



To find the area of a **compound shape**, draw lines to split the shape into simple shapes. Find the area of each shape separately. Add to find the total area.



The **volume** of a 3D solid is the amount of space it takes up. Volume is measured in  $\text{mm}^3$ ,  $\text{cm}^3$  or  $\text{m}^3$ .

Volume of a prism = area of cross-section  $\times$  length

# Graphs (Unit 9 Foundation)

## Co-ordinates

These are given in the form (X,Y). We go along the x axis and up or down the y axis.

## Y intercept

This is the point where the line crosses the y axis. On the example the y intercept = +2

## Gradient

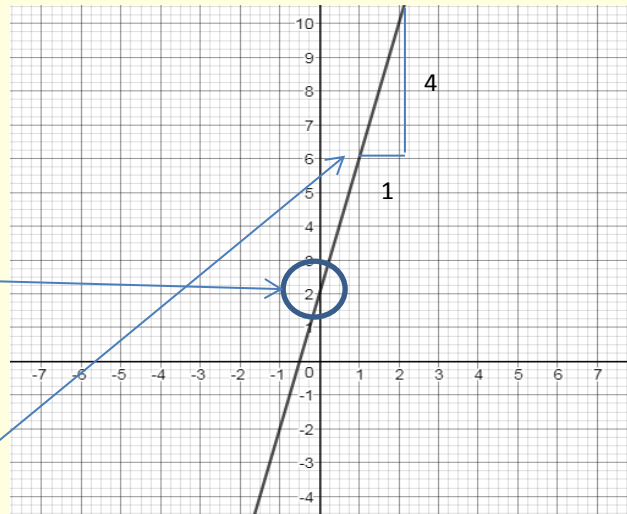
The steepness of a graph is called the **GRADIENT**. You can find the gradient by :

Squares up or down  
Squares across

$$\frac{4}{1}$$

Gradient + 4

Gradient can be positive (/) or negative (\)



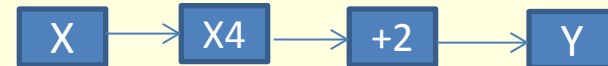
**Parallel Lines** have the same gradient but a different y intercept. For example a parallel line for the above graph would be  $y = 4x - 3$

**Mid points** is the point exactly in the middle. To find the coordinates add the x coordinates together and divide by 2 and do the same for the y coordinates.

## Table of Values/ Plotting graphs

To find the coordinates of a straight line you can use a table of values.

Firstly create a function machine



Then input numbers from the x axis to find the y axis.

These create coordinates which you can then plot onto the graph and join up with a ruler.

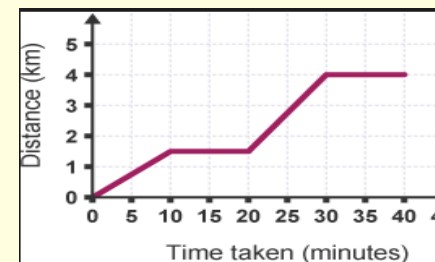
X	0	1	2	3
Y	2	6	10	14

## Distance time graphs

Represents a journey. The vertical axis represents the distance from starting point. The horizontal line represents time taken.

A horizontal line on a distance time graph represents an object at rest.

The gradient of the line represents the speed of the journey



$$Y = mx + c$$

Gradient

Y intercept

You can use the gradient and y intercept to write an equation for a line.  
Equation for above line is  $y = 4x + 2$

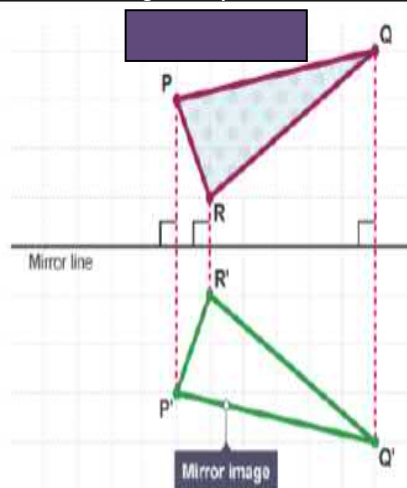
# Transformations (Unit 10 Foundation)

## Reflection

Every point in the image is the same distance from the mirror line as the original shape.

The line joining a point on the original shape to the same point on the image is perpendicular to the mirror line.

A reflection creates a congruent image



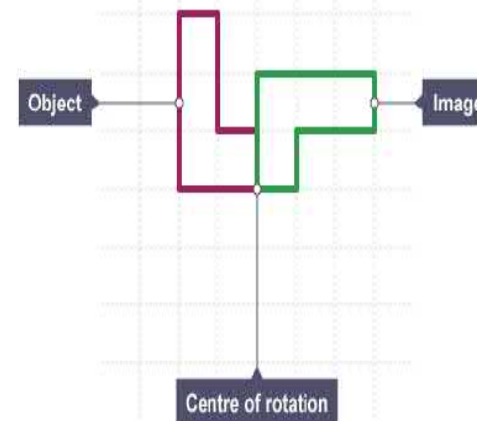
## Rotation

**Rotation** turns a shape around a fixed point called the **centre of rotation**.

3 parts of a rotation

- the centre of rotation
- the angle of rotation
- the direction of rotation

A Rotation creates a congruent image



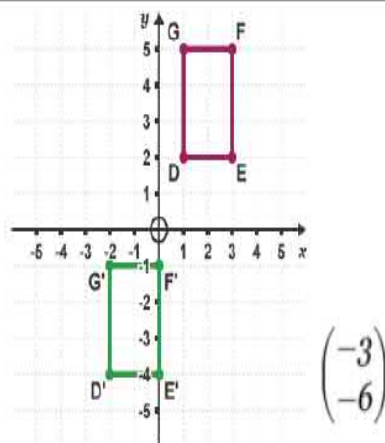
## Translation

A **translation** moves a shape up, down or from side to side and creates a congruent image.

Column vectors are used to describe translations

$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$  means translate the shape 4 squares to the right and 3 squares down.

$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$  means translate the shape 2 squares to the left and 1 square up.



## Enlargement

**Enlarging** a shape changes its size

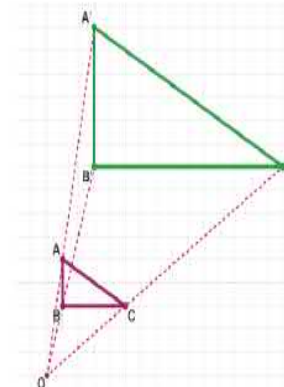
2 parts of an enlargement

- the scale factor
- the centre of enlargement

Fractional SF reduces the shape

Negative SF inverts the shape

An enlargement creates a similar shape



ABC has been enlarged by sf 3 about O.

## Linked Prior Topics

Shapes  
Scales  
Angles  
Straight line graphs

## Vocabulary

Object – Starting shape  
Image – Created by a transformation  
Congruent – 2 shapes are exactly the same  
Similar – 2 shapes with the same angles but different length sides  
Perpendicular – Forms a 90° angle

## Linked Future Topics

Transformation of functions  
Similar shapes

# RATIO

This is used to compare two or more amounts  
*Always draw boxes when dealing with ratio!*

## Writing a Ratio

The amount of one object compared with another. Eg there are 2 triangles to 5 squares

**2:5**



## Simplifying a Ratio [Video 269](#)

You simplify a ratio by dividing the numbers by the HCF (Highest Common Factor)

Simplify 6:12

Divide both by 6

1:2

Simplify 3:9:15

Divide all numbers by 3

1:3:5

Simplify 6:1.5

Multiply both sides by 2

12:3

Divide both sides by 3

4:1

# Unit 11 Foundation Ratio & Proportion

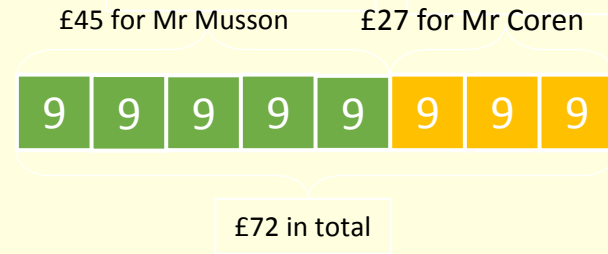
## Sharing an Amount in a Ratio

[Video 270](#)

Mr Musson and Mr Coren get **£72** pocket money.  
 They share it in the ratio **5:3**.

Draw a total of 8 boxes (5 + 3 = 8)  
 Split the money evenly between each box ( $72 \div 8 = 9$ )

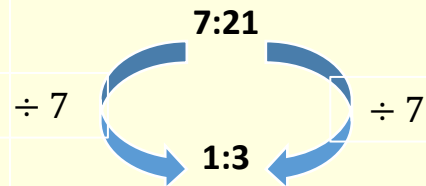
Mr Musson gets 5 boxes =  $5 \times 9 = £45$   
 Mr Coren gets 3 boxes =  $3 \times 9 = £27$



## Writing in the Ratio 1:n

You need to divide **both** sides by the **same** amount until the correct number is down to 1

Write **7:21** in the ratio **1:n**



# PROPORTION

*Proportion compares a part with a whole*



[Video 210](#)

## Best Buy

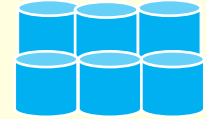
Video

This is about finding which item is better **value for money**

### Example 1



A pack of 4 tins of baked beans cost £1.96



A pack of 6 tins of baked beans cost £3

*Hint: Find the cost of **one** tin from each pack*

$$\begin{aligned} \pounds 1.96 \div 4 &= \pounds 0.49 \\ &= 49\text{p per tin} \end{aligned}$$

$$\begin{aligned} \pounds 3 \div 6 &= \pounds 0.50 \\ &= 50\text{p per tin} \end{aligned}$$

**Therefore the pack of 4 tins is better value for money**

### Example 2

Radox hand wash is on sale at Boots and Superdrug

Boots  
 500ml bottle costs £2.24

Superdrug  
 200ml bottle costs 90p

*Hint: multiply both to the **same** amount of hand wash*

$$\begin{aligned} \times 2 \quad \left( \begin{array}{l} 500\text{ml} = \pounds 2.24 \\ 1000\text{ml} = \pounds 4.48 \end{array} \right) \times 2 \end{aligned}$$

$$\begin{aligned} \times 5 \quad \left( \begin{array}{l} 200\text{ml} = 90\text{p} \\ 1000\text{ml} = \pounds 4.50 \end{array} \right) \times 5 \end{aligned}$$

**Therefore the bottle from boots is better value for money**

This is an example of DIRECT PROPORTION

## Pythagoras Theorem

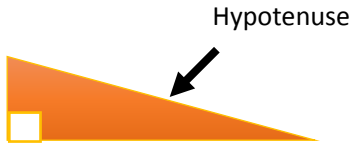
$$a^2 + b^2 = c^2$$

Pythagoras is used to find missing sides in **Right-angled triangles**

### Key Facts

#### HYPOTENUSE

This is the longest side in a right-angled triangle and is **ALWAYS** opposite the right angle



### Method to find the hypotenuse

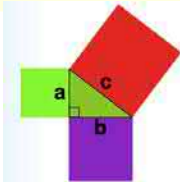
[Pythagoras video 257](#)

- Square side a
- Square side b
- Add together
- Square root

$$a^2 + b^2 = c^2$$

### Method to find a shorter side

- Square side c
- Square side a/b (whichever is known)
- Subtract a/b from c
- Square root



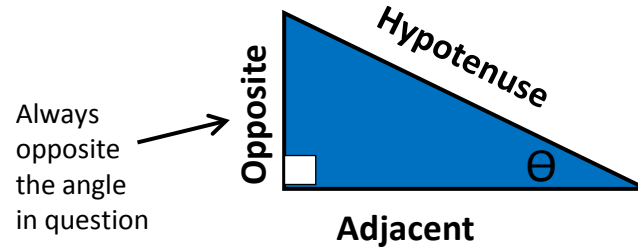
## Unit 12 Foundation Right-Angled Triangles 1

[Trigonometry Video 329, 330, 331](#)

### Trigonometry

Used to find missing sides and angles in right-angled triangles

You must label your sides correctly



Using the Tangent Ratio:

$$Opp = \tan \theta \times Adj$$

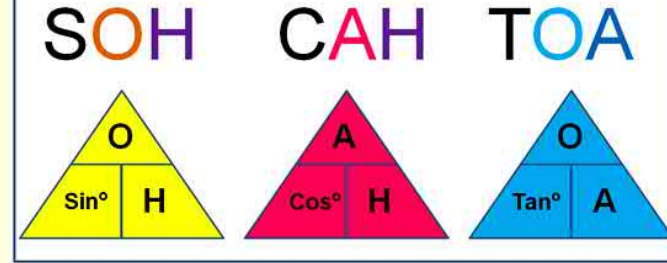
$$Adj = \frac{Opp}{\tan \theta}$$

$$\tan^{-1} \theta = \frac{Opp}{Adj}$$

Press shift and then tan on your calculator

## SOH – CAH – TOA Pyramids

Cover the letter which is the unknown value, and then Multiply for horizontal relationships and Divide for vertical relationships



Using the Sine Ratio

$$Opp = \sin \theta \times Hyp$$

$$Hyp = \frac{Opp}{\sin \theta}$$

$$\sin^{-1} \theta = \frac{Opp}{Hyp}$$

Press shift and then sin on your calculator

Using the Cosine Ratio:

$$Adj = \cos \theta \times Hyp$$

$$Hyp = \frac{Adj}{\cos \theta}$$

$$\cos^{-1} \theta = \frac{Adj}{Hyp}$$

Press shift and then cos on your calculator



## Unit 13 Foundation

The **probability** of something (let's call it outcome **A**) happening is written as **P(A)**, and must be between 0 and 1.  
 If **P(A) = 0**, it is impossible.  
 If **P(A) = 1**, it is certain to happen.

The probability of A not happening is written as **P(A')**. Since A will either happen or not happen,  
**P(A) + P(A') = 1** [Video 250: Events not happening](#)

We call the two outcomes above "mutually exclusive" – this means they cannot happen at the same time. The probabilities of all possible outcomes for an event always add up to 1, because one of them is certain to happen.

**Example:**  
 Event: rolling **3** on a fair six-sided dice.

$P(3) = 1/6$   
 $P(3') = 5/6$  ← These two outcomes are mutually exclusive and cover every possibility, so their probabilities add up to 1

[Video 249: Independent Events](#)

If the outcome of one event doesn't affect the outcome of another, we call those events independent. For example, **flipping a coin** and **rolling a dice** are independent of each other.

**Experimental probability** is about estimating probability based on previous outcomes, (unlike **theoretical** probability, which was used above and is based on what should happen). Experimental probability would be written as [Video 248: Relative Frequency](#)

$\frac{\text{frequency of desired outcomes}}{\text{total number of trials}}$  ← "**Trials**" refers to what you actually do for your experiment (flipping a coin; counting cars as they drive past). Each time you do it counts as one trial.

**Example:**  
 Experiment: spinning a fair four-numbered spinner 100 times (i.e. 100 trials)

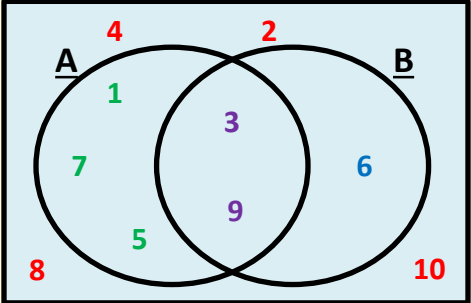
Score	1	2	3	4
Frequency	23	26	30	21

Based on these results, the **P(1) = 23/100**, or **0.23**. To estimate relative frequency, multiply the number of intended trials by the experimental probability, e.g. for **200 trials**, we would predict **46** results will be 1 because **200 x 0.23 = 46**, and **60** results will be 3 because **200 x 0.30 = 60**.

**Venn diagrams** show how two or more **sets** (groups) can overlap, and we can use them to calculate the probability of a given **element** (item in a set) being chosen. They can have each element individually written in them ([Example 1](#)), or just the quantity of each section ([Example 2](#)). The **universal set** ( $\xi$ ) contains **everything** being considered.

**Example 1**

$\xi$ : Integers up to 10  
**A**: Odd numbers  
**B**: Multiples of 3

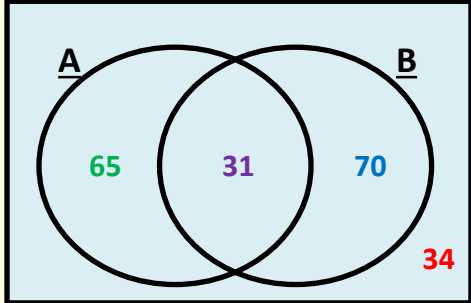


$A \cap B$ : **only** elements in both **A** and **B**  
 $A \cup B$ : **all** elements in **A** or **B** or **both**

If picking a number at random,  
 $P(A \cap B') = 3/10$      $P(A' \cap B) = 1/10$   
 $P(A \cap B) = 2/10$      $P(A' \cap B') = 4/10$

**Example 2**

$\xi$ : Year 10 students (200 in total)  
**A**: Students who walk to school  
**B**: Students who like football



$A \cap B'$ : elements in **A** and **not** in **B**  
 $A' \cap B$ : elements in **B** and **not** in **A**

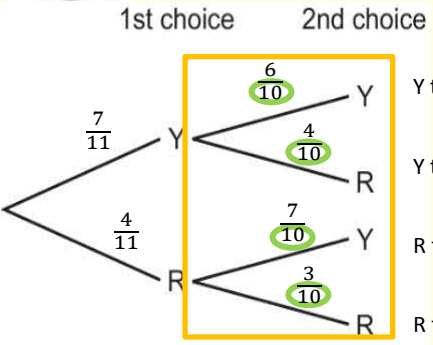
If picking a student at random,  
 $P(A \cap B') = 65/200$      $P(A' \cap B) = 70/200$   
 $P(A \cap B) = 31/200$      $P(A' \cap B') = 34/200$

[Video 380: Venn Diagrams](#)

**Probability "tree" diagrams** show the possible outcomes of multiple events one after the other. The "branches" are for each outcome and every set of branches adds to 1.



There are 11 balls in the bag, so the first choice is out of 11. The second choice is out of 10, since a ball has been taken out, so the denominators change



The second choice has two sets of branches because there are two possible scenarios for it (either after a yellow or after a red).

Y then Y:  $P(YY) = \frac{7}{11} \times \frac{6}{10} = \frac{42}{110}$   
 Y then R:  $P(YR) = \frac{7}{11} \times \frac{4}{10} = \frac{28}{110}$   
 R then Y:  $P(RY) = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$   
 R then R:  $P(RR) = \frac{4}{11} \times \frac{3}{10} = \frac{12}{110}$

To find the final probability, multiply along the branches as shown.

(For example, the probability of picking red both times is **12/110**).

[Video 252: Tree Diagrams](#)

# Unit 14 Foundation Multiplicative Reasoning

$$0.1 = \frac{1}{10} = 10\% \quad 0.01 = \frac{1}{100} = 1\%$$

The original amount of something is always 100%, if it is increased then the final amount is more than 100%. If it is decreased then it is less than 100%.

To find out a percentage of something, convert the % to a decimal and multiply

e.g. Find 80% of 45  
 $0.8 \times 45 = 36$

[V233](#)

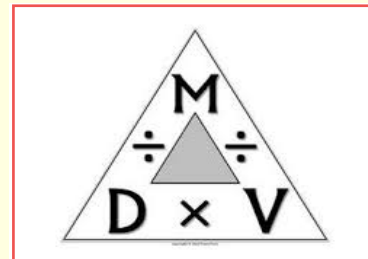
To calculate percentage change:

$$\text{Percentage change} = \frac{\text{actual change}}{\text{original amount}} \times 100$$

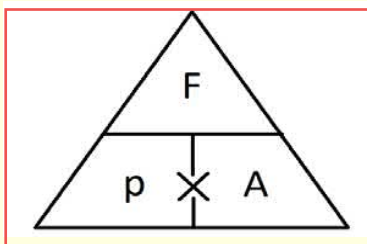
*Compound vs simple interest:* in compound interest, you are paid interest on the amount. The second time interest is paid, it is paid on the *original* amount *and* the interest added before  
 Calculating compound interest:  $\text{Final amount} = \text{initial amount} \times \text{interest rate}^{\text{time}}$

[V236](#)

*Density:* is a measure of the mass of a substance contained in a certain volume  
 Usually measured in  $\text{g/cm}^3$



[V384](#)



*Pressure:* is the force applied per unit area

Pressure is usually measured in  $\text{N/m}^2$

[V385](#)

*Kinematics Formulae:* these are also called equations of motion, but you don't need to learn them in Maths.

$v$  = final speed,  $u$  = initial speed,  $a$  = acceleration,  $t$  = time,  $s$  = distance

$$\begin{aligned} v &= u + at \\ s &= ut + \frac{1}{2}at^2 \\ v^2 &= u^2 + 2as \end{aligned}$$

*Velocity vs speed:* velocity is speed in a certain direction (often measured in  $\text{m/s}$ )

*Acceleration:* is the rate of change of velocity (often measured in  $\text{m/s}^2$ )

*Direct Proportion:* if two quantities are in direct proportion, as one increases, the other increases by the same percentage

$$y \propto x$$

$$y = kx, \text{ where } k \text{ is a constant value}$$



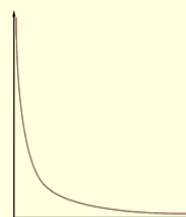
**DIRECT PROPORTION**

[V254](#)

*Indirect Proportion:* is when one value increases as the other value decreases.

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$



**INVERSE PROPORTION**

[V255](#)

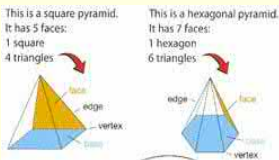
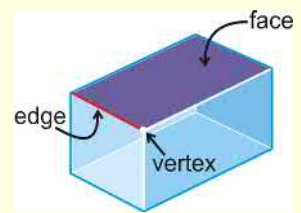


# Unit 15 Foundation: Constructions, Loci and Bearings

**Face:** the flat edge of a 3D shape **Edge:** the lines where two faces meet

**Vertex (pl. vertices):** the corners that edges meet at

V5

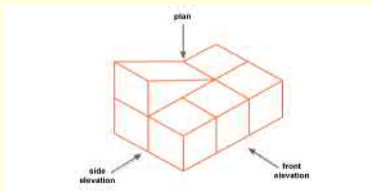
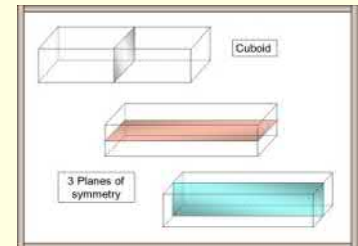


**Pyramids:** have a base that can be any shape and sloping triangular sides that meet at a point

**Right prism:** the sides are at right angles (perpendicular)

**Plane:** is a flat surface

**Plane of symmetry:** is when a plane cuts a shape in half so that the part on one side of the plane is identical to the other



**Plan:** is the view from above an object

**Front elevation:** is the view of the front of an object

**Side elevation:** is the view from the side of an object

**Drawing an accurate triangle:** you can draw this with a ruler and protractor if you know three measurements (length of 2 sides and 1 angle OR length of 1 side and 2 angles)

V81

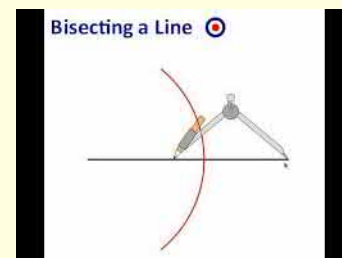
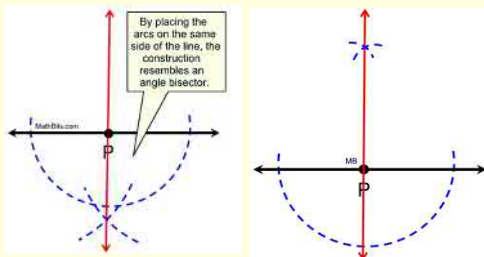
V82

V83

**Scale:** A scale is a ratio that shows the relationship between a drawn length and a real length, e.g. on a map.

**Constructions:** Are accurate drawings made using a pair of compasses.

**Bisecting a line:** Means to cut exactly in half



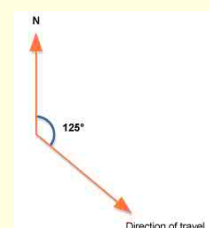
V78

**Angle bisector:** Cuts an angle exactly in half.

V72

**Locus (pl. loci):** A set of all points that obey a given rule. A locus creates a bounded region.

**Bearing:** Is an angle measured in degrees clockwise from north.



## Expanding and factorising quadratics (double brackets)

Expanding a quadratic is just like multiplying 2-digit numbers – use a multiplication grid, then add your answers:

$$(x + 2)(x + 3) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$$

x	( x	+2 )
( x	x <sup>2</sup>	+2x
+3	+3x	+6

It's no coincidence!

[Video 14: Expanding quadratics](#)

## Plotting a Quadratic Graph

To plot a quadratic, make the expression equal to y, then make a table using different values of x. For example:

$$y = x^2 - 4x + 5$$

If  $x = 1, y = (1)^2 - (4 \times 1) + 5$

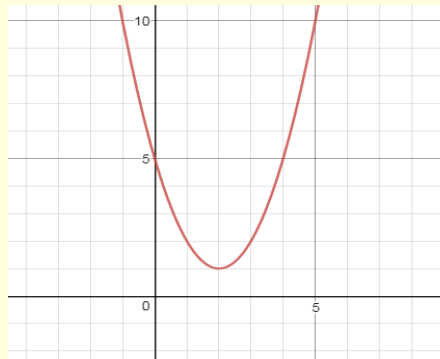
If  $x = 1, y = 2$

x	0	1	2	3	4
y	5	2	1	2	5

[Video 264: Plotting a quadratic graph](#)

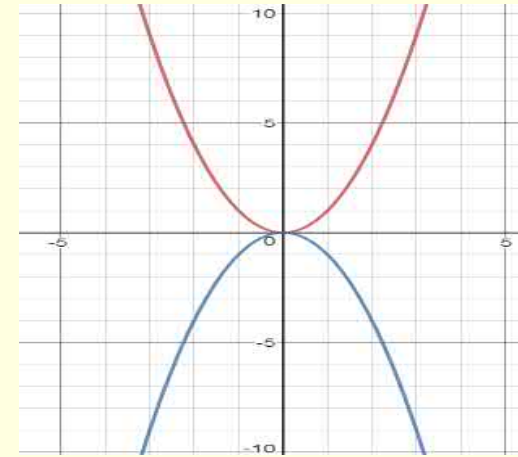
[Video 265: Sketching a quadratic graph using key coordinates](#)

Based on the table above, the coordinates to plot would be: **(0, 5) (1, 2) (2, 1) (3, 2) (4, 5)**



## Recognising a quadratic shape

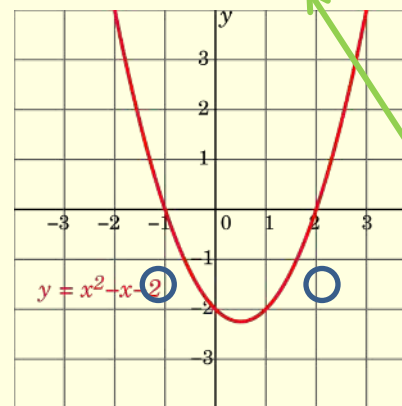
All  $y = x^2$  graphs will have the same symmetrical curved shape you see below, even if you can't see all of it. At any point on the line, the y-coordinate is the **square** of the x-coordinate



The upside down graph shows the equation  $y = -x^2$ , which is just the reflection of the positive version (the y-values have all become negative).

On the diagram, the solutions are **-1** and **2** (circled), because that's where  $y = 0$ .

Some quadratics **(like the one over there)** do not cross the x-axis. This means they have no "solutions", because the y value never reaches 0!



[Video 267: Using the quadratic formula](#)

when  $x^2 + bx + c = 0$ ,  $x = \frac{-b \pm \sqrt{4c}}{2}$

Factorising a quadratic is the opposite of expanding it – you're putting it back into brackets (if you can). You can still use the grid, but do it in reverse:

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

x	( x	+3 )
( x	x <sup>2</sup>	+3x
+4	+4x	+12

We know from expanding that the two numbers in my brackets will add to make 7, and multiply to make 12, so they must be 3 and 4 ( $3x + 4x = 7x$  and  $3 \times 4 = 12$ )

[Video 118: Factorising quadratics](#)

## Solving quadratics

Quadratic equations are written as equal to y, like so:

$$y = x^2 + bx + c$$

To find the solutions, we make them equal to 0 because the "solutions" are the "x-intercepts", where the graph crosses the x-axis. On the x-axis, the y-value would be zero (because we haven't moved up or down).

$$x^2 + 7x + 12 = 0$$

Then we can factorise to give two answers (one of the brackets must = 0).

$$(x + 3)(x + 4) = 0$$

$$x + 3 = 0 \text{ or } x + 4 = 0$$

$$x = -3 \text{ or } x = -4$$

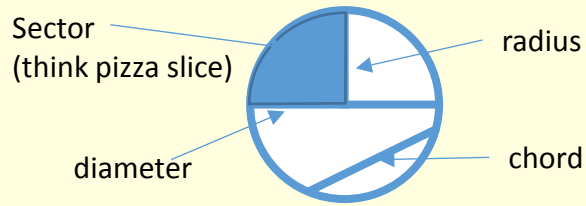
[Video 266: Solving quadratics by factorising](#)

If we can't factorise (sometimes the numbers don't work), we can use the **quadratic formula**:

[Video 61](#)

### Key Facts

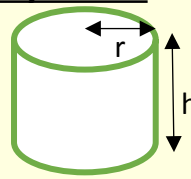
Circumference = perimeter of a circle (units)  
 Area = space inside a 2D shape (units<sup>2</sup>)  
 Volume = the space inside a 3D shape (units<sup>3</sup>)



## Perimeter, Area and Volume 2

### Volume and SA of cylinders

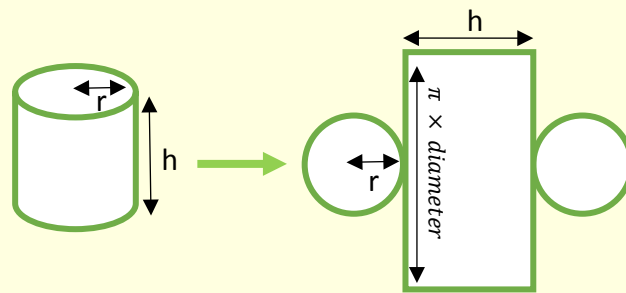
**Volume =  $\pi r^2 h$**   
 $V = \pi \times \text{radius}^2 \times \text{height}$



(this is just the area of one of the circles multiplied by how long your cylinder is)

### Surface Area

[Volume = video 357](#)



SA = 2 circle areas + rectangle area  
**SA =  $2\pi r^2 + \pi dh$**

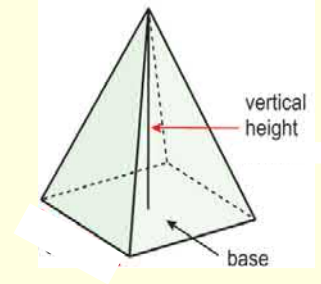
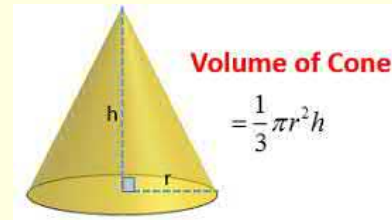
[SA = video 315](#)

### Volume of pyramids and cones

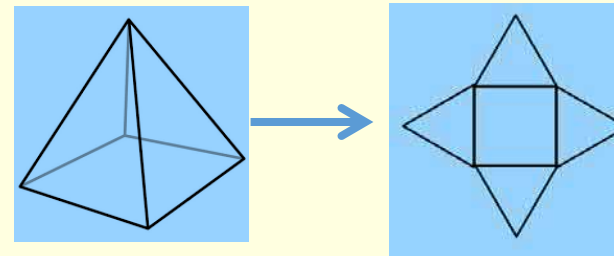
**Volume of a Pyramid/Cone =  $\frac{1}{3} \times \text{area of base} \times \text{vertical height}$**

[Volume of cone = video 359](#)

[Volume of pyramid = video 360](#)



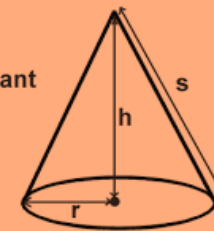
### Surface Area of a Pyramid = total area of all faces



Area of all 4 triangles + area of the base

**Surface Area of a Cone =  $\pi \times \text{radius} \times \text{slant height}$**   
 $= \pi r l$

r = radius  
 h = height  
 s = length of slant



[SA of Cone = video 314](#)

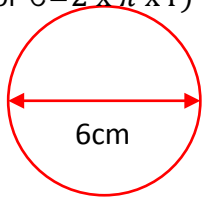
[Circumference = video 60](#)

[Area = Video 40](#)

### Circumference and Area of Circles

**Circumference =  $\pi \times \text{diameter}$**  (or  $C = 2 \times \pi \times r$ )

**Area =  $\pi \times \text{radius}^2$**



" $\pi r^2$  sounds like area to me, if you need the circumference you just use  $\pi d$ "

Remember our radius is half of our diameter

[Arc length = video 58](#)

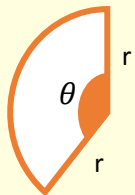
### Semicircles and Sectors

#### Perimeter

**Perimeter = arc length + radius + radius**

**Arc length =  $\frac{\theta}{360} \times \pi \times \text{diameter}$**

Arc length is a fraction of the circumference



#### Area

[Video 46](#)

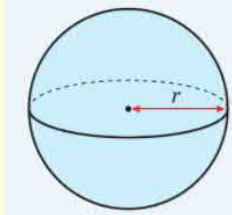
**Area =  $\frac{\theta}{360} \times \pi \times \text{radius}^2$**

[Perimeter of semi-circle = video 62](#)

### Volume and surface area of spheres

**Volume of Sphere =  $\frac{4}{3} \pi r^3$**

**Surface Area of a Sphere =  $4\pi r^2$**



[SA = video 313](#)

[Volume = video 361](#)

## Power of 0

Anything to the power of 0 is equal to 1, no matter what it is! We can show this by dividing two identical indices:

$$3^2 \div 3^2 = 3^{(2-2)}$$

$$3^2 \div 3^2 = 3^0$$

Since dividing a value by itself always gives the answer 1, we also know that:

$$3^2 \div 3^2 = 1, \text{ therefore } 3^0 = 1$$

**This works for all numbers AND letters!**

## Combining indices

When multiplying indices with the same base value, **add the powers**:

$$2^2 \times 2^3 = \underline{2 \times 2} \times \underline{2 \times 2 \times 2}, \text{ so}$$

$$2^2 \times 2^3 = 2^{(2+3)} = 2^5$$

[Video 174: Laws of indices \(including power of 0\)](#)

When dividing indices with the same base value, **subtract the powers**:

$$3^6 \div 3^2 = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$$

$$3^6 \div 3^2 = 3^{(6-2)} = 3^4$$

## Multiplying and dividing fractions

To multiply fractions, just multiply the **numerators** and multiply the **denominators** (then simplify if you can!)

$$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15} (= \frac{2}{5})$$

[Multiplying fractions](#)

To divide by a fraction, multiply by the **reciprocal** (flip the numerator and denominator)

[Dividing fractions](#)

$$\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{2 \times 5}{3 \times 3} = \frac{10}{9}$$

## Reciprocals

When two numbers are reciprocal, it means they **multiply to make 1** (they're a bit like "opposites").

So 2 and ½ are reciprocal because  $2 \times \frac{1}{2} = 1$

Reciprocal fractions are the **reverse** of each other, as shown:

$$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1$$

The numerator will always match the denominator, and we know that anything divided by itself is 1!

## Negative indices

Raising something to a negative power is the same as raising the **reciprocal** (see left) to the positive power.

[Video 175: Negative indices](#)

$$3^{-2} = \left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} = \frac{1}{3^2}$$

It's no coincidence!

$$\text{Negative power} = \frac{1}{\text{positive power}}$$



## Standard form

Standard form is a way of writing very large or very small numbers using powers of 10 (multiplying/dividing by 10 until the decimal point is in the right place). The base number must always be between 1 and 10.

[Video 300: Standard form](#)

e.g. 50000000000000000000  
can be written as  
 $5 \times 1000000000000000000$ ,  
which can then be written as  
 $5 \times 10^{18}$   
Clearly, the last way is quicker!

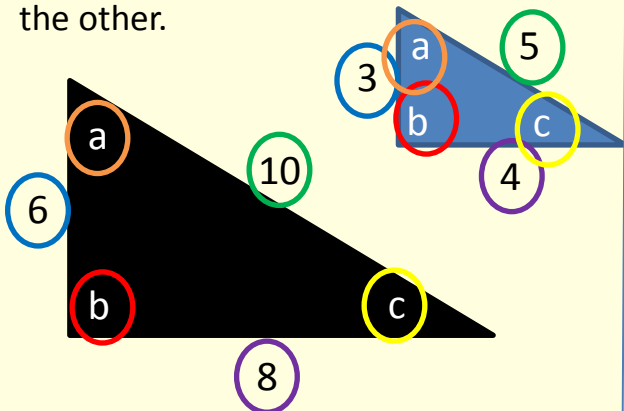
$$0.000000000004 = 4 \div 100000000000$$

$$= 4 \times 10^{-11}$$

**NOTE:**  
When we divide, we use negative powers!

## SIMILARITY

When shapes look the same but are different sizes, they are mathematically **similar**. This means their **corresponding** ("matching") **angles** are **equal**, and their **corresponding sides** are in the **same ratio**. One shape is an *enlargement* of the other.



[Congruence & Similarity definitions](#)  
[How to find missing sides](#)

## VECTORS

### Unit 19 Foundation

Column vectors describe horizontal and vertical "movement", a bit like how co-ordinates describe position. They look similar, but they're arranged in a column (hence the name), as shown below:

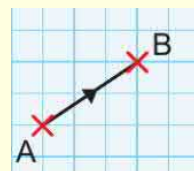
[Column vectors](#)

$\begin{pmatrix} x \\ y \end{pmatrix}$  horizontal movement  
 vertical movement

To get from A to B, you go 3 right, 2 up:

$$\vec{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\text{Reverse: } \vec{BA} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$



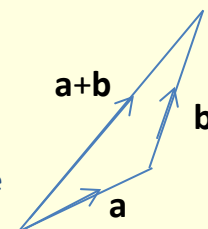
Vectors are labelled with a lower case letter, either **bold** or underlined.

You can combine vectors by adding their x and y values to give a **resultant** vector:

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad \mathbf{a+b} = \begin{pmatrix} 3+4 \\ 2+1 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

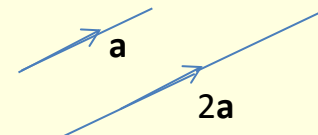
It would look like this:

We do this to move between points that don't have a vector between them – you can only go the way you know!

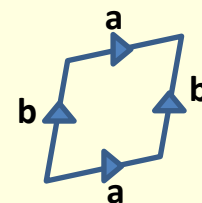


Vectors can also be multiplied:

$$2\mathbf{a} = \begin{pmatrix} 3 \times 2 \\ 2 \times 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$



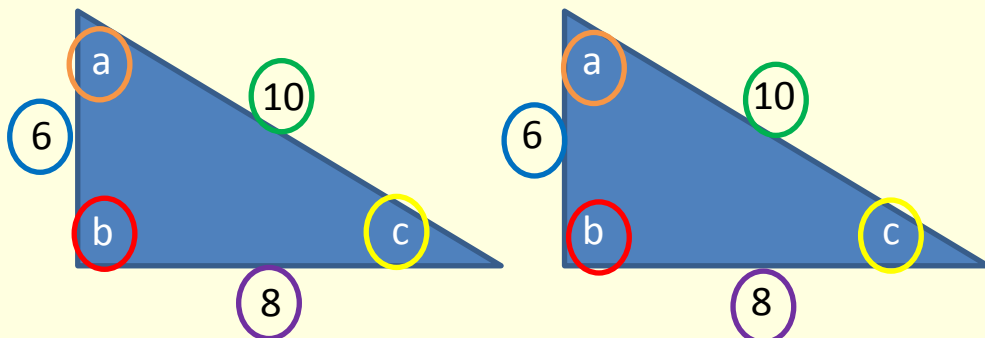
Parallel vectors can be represented using the same letter:



[Algebraic vectors](#)

## CONGRUENCE

When shapes are identical, they are **congruent**. All **corresponding** lengths and angles are **equal** – you could fit one perfectly on top of the other.



You can prove two triangles are congruent by showing that any of these combinations are matching ([video here](#)):

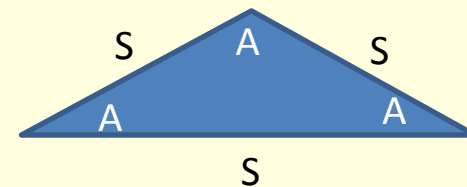
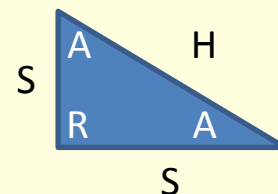
SSS (all three sides)

SAS (two sides and the angle between them)

ASA (two angles and the side which connects them)

AAS (two angles and the side after the second angle)

RHS (right angle, hypotenuse and one other side)\*



\*only applies to right-angled triangles

## Unit 20 Foundation More Algebra

**Quadratic functions** contain a term in  $x^2$  but no higher power of  $x$ .

[Video 266 - https://tinyurl.com/y8san5jm](https://tinyurl.com/y8san5jm)

**Cubic functions** contain a term in  $x^3$  but no higher power of  $x$ .

[Video 344 - https://tinyurl.com/yamclpto](https://tinyurl.com/yamclpto)

Cubic functions can contain terms in  $x^2$ ,  $x$ , and number terms.

When a cubic function is equal to zero it may have one, two, or three solutions. The solution to a cubic function equalling zero is where the graph crosses the  $x$ -axis. The solutions are commonly called **roots**.

[Video 264 - https://tinyurl.com/y7u3d79a](https://tinyurl.com/y7u3d79a)

The **reciprocal** function ( $y = \frac{1}{x}$ ) of a cubic function has the  $x$ - and  $y$ -axes as **asymptotes** to the graph.

[Video 346 - https://tinyurl.com/yd8x2uz8](https://tinyurl.com/yd8x2uz8)

An asymptote is a line that the graph gets closer and closer to, but never actually touches.

When a graph has  $x$  and  $y$  in **direct proportion**,  $y = kx$

[Video 254 - https://tinyurl.com/htma465](https://tinyurl.com/htma465)

When a graph has  $x$  and  $y$  **inversely proportional** to each other,  $y = \frac{k}{x}$

[Video 255 - https://tinyurl.com/yb2ur2yq](https://tinyurl.com/yb2ur2yq)

The graph of two quantities that are inversely proportional is a reciprocal graph.

**Simultaneous equations** are equations that are both true for a pair of variables (letters).

[Video 296 - https://tinyurl.com/y9dbmoe](https://tinyurl.com/y9dbmoe)

Simultaneous equations can be solved graphically by plotting both equations on the same coordinate grid. The point at which the lines cross (the point of **intersection**) has the coordinates that are the solution.

Simultaneous equations can also be solved by the elimination method. To do this, the coefficients of either the  $x$  or  $y$  terms must be equal (or equal with the opposite sign).

[Video 295 - https://tinyurl.com/yadevfgk](https://tinyurl.com/yadevfgk)  
Subtract (or add) the two equations to eliminate one of the terms. The remaining term can now be evaluated.

The **subject** of a formula is the letter on its own side of the equals sign.

[Video 7 - https://tinyurl.com/yc6vax5f](https://tinyurl.com/yc6vax5f)  
You can change the subject of a formula using **inverse operations** (subtract to move an added term to the other side, etc).

[Video 8 - https://tinyurl.com/yahmeoyn](https://tinyurl.com/yahmeoyn)  
An even number is a multiple of 2.  $2m$  and  $2n$  are general terms for even numbers where  $m$  and  $n$  are integers.

An **equation** has an equals sign ( $=$ ). You can solve it to find one value of the letter (unknown/variable).

An **identity** has an equivalent (triple bar) sign ( $\equiv$ ). The left hand side equals the right hand side for all values of the letter (unknown/variable).

Key Points:



<https://tinyurl.com/ybfxnjsj>

Knowledge Check:



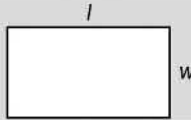
<https://tinyurl.com/y9nl3tka>

# Edexcel GCSE (9-1) Maths: need-to-know formulae

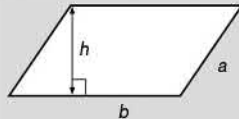
www.edexcel.com/gcsemathsformulae

## Areas

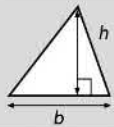
Rectangle =  $l \times w$



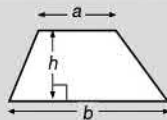
Parallelogram =  $b \times h$



Triangle =  $\frac{1}{2} b \times h$



Trapezium =  $\frac{1}{2} (a + b)h$

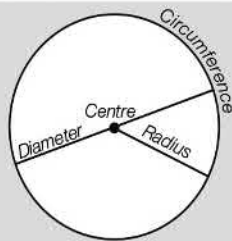


## Circles

Circumference =  $\pi \times \text{diameter}$ ,  $C = \pi d$

Circumference =  $2 \times \pi \times \text{radius}$ ,  $C = 2\pi r$

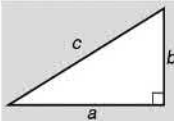
Area of a circle =  $\pi \times \text{radius squared}$ ,  $A = \pi r^2$



## Pythagoras

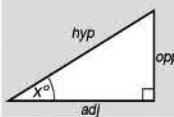
### Pythagoras' Theorem

For a right-angled triangle,  
 $a^2 + b^2 = c^2$



### Trigonometric ratios (new to F)

$\sin x^\circ = \frac{\text{opp}}{\text{hyp}}$ ,  $\cos x^\circ = \frac{\text{adj}}{\text{hyp}}$ ,  $\tan x^\circ = \frac{\text{opp}}{\text{adj}}$



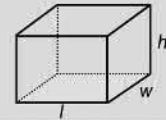
## Quadratic equations

### The Quadratic Equation

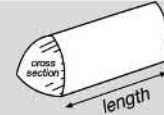
The solutions of  $ax^2 + bx + c = 0$ ,  
where  $a \neq 0$ , are given by  $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

## Volumes

Cuboid =  $l \times w \times h$



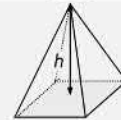
Prism = area of cross section  $\times$  length



Cylinder =  $\pi r^2 h$



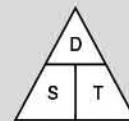
Pyramid =  $\frac{1}{3} \times \text{area of base} \times h$



## Compound measures

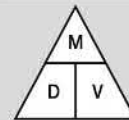
### Speed

speed =  $\frac{\text{distance}}{\text{time}}$



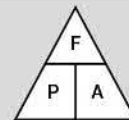
### Density

density =  $\frac{\text{mass}}{\text{volume}}$



### Pressure

pressure =  $\frac{\text{force}}{\text{area}}$

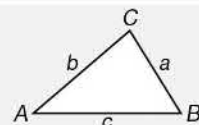


## Trigonometric formulae

Sine Rule  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule  $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle =  $\frac{1}{2} ab \sin C$



Foundation tier formulae

Higher tier formulae

U679



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