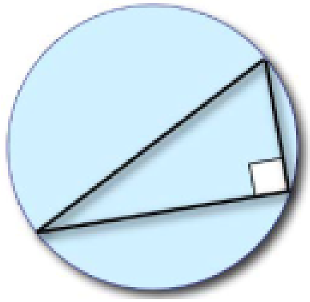
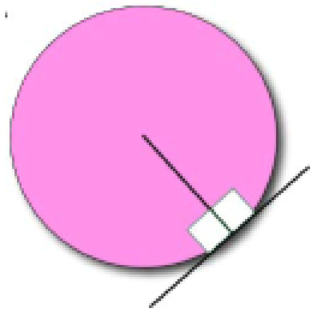
V64 V65



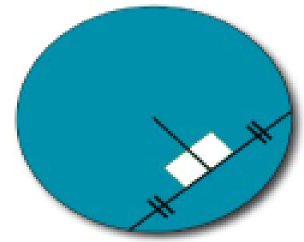
The angle in a semicircle is a right angle.

★ V65a



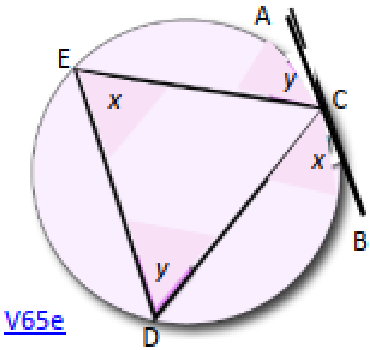
The angle between a **tangent** and **radius** is a right angle.

V65f



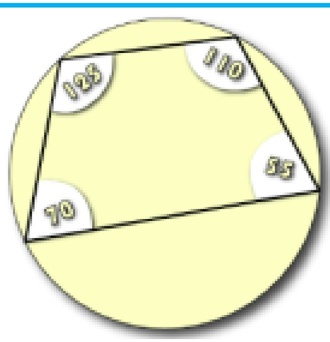
A **chord** is a straight line connecting two points on a circle.

The **perpendicular** from the centre of the circle to a chord **bisects** the chord and the line drawn from the centre of the circle to the **midpoint** of a chord is at right angles to the chord.



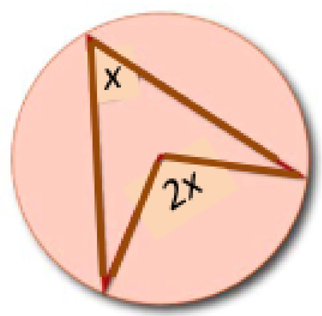
V65e

AB is a tangent to the circle. CD, DE and CE are **chords**. Angle ECA is the angle between the **tangent** and the chord in one segment. The other **segment** has angle CDE. This is the **alternate segment.** The angle between the chord and tangent is equal to the angle in the alternate segment.



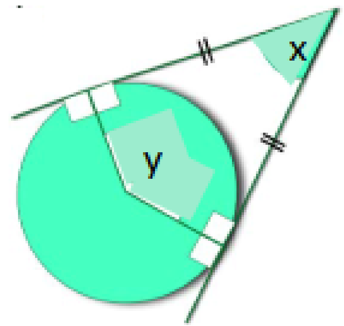
A cyclic quadrilateral with all four vertices on the circumference of the circle. Opposite angles add up to 180°.

★ V65d

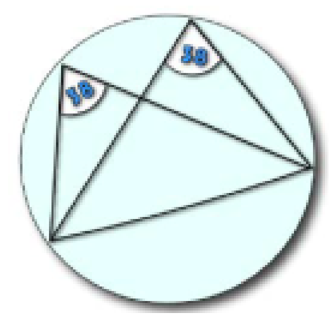


The angle at the centre of a circle is twice the angle at the circumference when both are subtended by the same arc.

★ V65b



Tangents drawn from a point outside the circle are equal in length.
$x + y = 180$



Angles in the same segment and standing on the same chord are always equal.

★ V65c

Videos: V8 V21 V22 V23 V24 V365 V370

You can change the subject of a formula by isolating the terms involving the new subject.

When the letter to be made the subject appears twice in the formula you will need to factorise.

You may need to factorise before simplifying an algebraic fraction:

- Factorise the numerator and denominator.
- Divide the numerator and denominator by any common factors.

You may need to factorise the numerator and/or denominator before you multiply or divide algebraic fractions.

To add or subtract algebraic fractions, write each fraction as an equivalent fraction with a common denominator.

To find the lowest common denominator of algebraic fractions, you may need to factorise the denominators first.

To rationalise the fraction $\frac{1}{a \pm \sqrt{b}}$, multiply by $\frac{a \mp \sqrt{b}}{a \mp \sqrt{b}}$.

To show a statement is an identity, expand and simplify the expressions on one or both sides of the equals sign, until the two expressions are the same.

A function is a rule for working out values of y when given values of x e.g. $y = 3x$ and $y = x^2$

The notation f(x) is read as 'f of x'.

fg is the composition of the function f with the function g. To work out fg(x), first work out g(x) and then substitute your answer into f(x).

The inverse function reverses the effect of the original function. $f^{-1}(x)$ is the inverse of f(x).

To prove a statement is not true you can find a **counter-example** – an example that does not fit the statement.

For an algebraic proof let n represent any integer

Even number	$2n$
Odd number	$2n + 1$ or $2n - 1$
Consecutive numbers	$n, n + 1, n + 2, \ldots$
Consecutive even numbers	$2n, 2n + 2, 2n + 4, \ldots$
Consecutive odd numbers	$2n + 1, 2n + 3, 2n + 5, \ldots$

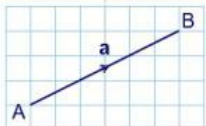
A **vector** is a quantity that has both size (or magnitude) and direction.

Examples of vector quantities are:
- displacement
- velocity
- force

A **scalar** is a quantity that has size (or magnitude) only.

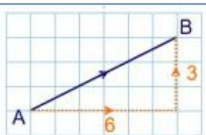
Examples of scalar quantities are:
- length
- speed

Vectors are written as **bold** lower case letters: **a**, **b**, **c**. When handwriting, underline the letter: a, b, c.



This vector goes from the point A to the point B.

We can write this vector as $\overrightarrow{AB}$.



To go from the point A to the point B we must move 6 units to the right and 3 units up.

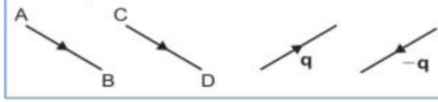
We can represent this movement using a **column vector**.

$\overrightarrow{AB} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

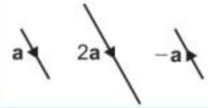
This is the horizontal component. It tells us the number of units in the x-direction.

This is the vertical component. It tells us the number of units in the y-direction.

If $\overrightarrow{AB} = \overrightarrow{CD}$ then the line segments AB and CD are equal in length and are parallel. $\overrightarrow{AB} = -\overrightarrow{BA}$



2**a** is twice as long as **a** and in the same direction.
−**a** is the same length as **a** but in the opposite direction.



Video 353 and 353a

With the origin O, the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are called the **position vectors** of the points A and B. In general, a point with coordinates (p, q) has position vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

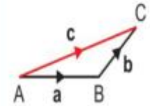
In general, if the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is multiplied by the scalar k, then

$$k \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

For example, $3 \times \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ 15 \end{pmatrix}$

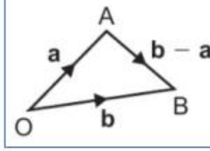
When a vector is multiplied by a scalar the resulting vector is either parallel to the original vector or lies on the same line.

Triangle law for vector addition: Let $\overrightarrow{AB}$ = **a**, $\overrightarrow{BC}$ = **b** and $\overrightarrow{AC}$ = **c**. Then **a** + **b** = **c** forms a triangle.

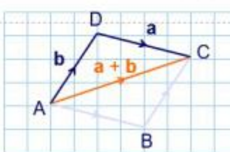


When **c** = **a** + **b** the vector **c** is called the **resultant vector** of the two vectors **a** and **b**.

When $\overrightarrow{OA}$ = **a** and $\overrightarrow{OB}$ = **b**, $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ = **b** − **a**.



Suppose **a** = $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and **b** = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$



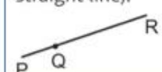
From this diagram we can see that

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ = **a** + **b**

Also

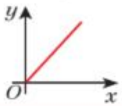
$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$ = **b** + **a**

$\overrightarrow{PQ} = k\overrightarrow{QR}$ shows that the lines PQ and QR are parallel. Also they both pass through point Q so PQ and QR are part of the same straight line. P, Q and R are said to be **collinear** (they all lie on the same straight line).



Videos: V345 V255 V254

When a graph of two quantities is a straight line through the origin, one quantity is directly proportional to the other.



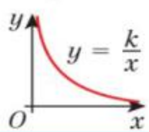
The symbol ∝ means 'is directly proportional to'.

If y is directly proportional to x, $y \propto x$ and $y = kx$, where k is a number, called the **constant of proportionality**.

Where k is the constant of proportionality:

- if y is proportional to the square of x then $y \propto x^2$ and $y = kx^2$
- if y is proportional to the cube of x then $y \propto x^3$ and $y = kx^3$
- if y is proportional to the square root of x then $y \propto \sqrt{x}$ and $y = k\sqrt{x}$

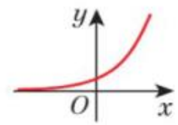
When y is **inversely proportional** to x, $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$



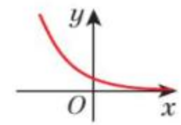
The tangent to a curved graph is a straight line that touches the graph at a point. The gradient at a point on a curve is the gradient of the tangent at that point.

Expressions of the form a^x or a^{-x}, where $a > 1$, are called **exponential functions**.

The graph of an exponential function has one of these shapes.



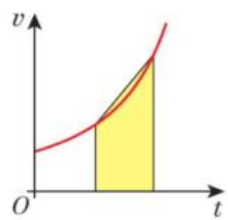
$y = a^x$ where $a > 1$ or
$y = b^{-x}$ where $0 < b < 1$
exponential growth



$y = a^{-x}$ where $a > 1$ or
$y = b^x$ where $0 < b < 1$
exponential decay

Exponential graphs intersect the y-axis at (0, 1) because $a^0 = 1$ for all values of a.

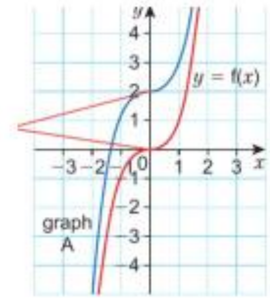
The area under a velocity–time graph shows the displacement, or distance from the starting point. To estimate the area under a part of a curved graph, draw a chord between the two points you are interested in, and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an estimate for the area under this part of the graph.

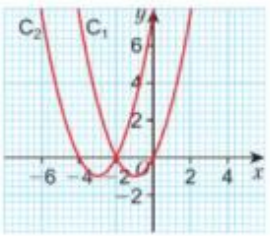


The gradient of the chord gives the average rate of change

Transformation of Graphs – Video Link

The graph of $y = f(x)$ is transformed into the graph of: $y = f(x) + a$ by a translation of a units parallel to the y-axis or a translation by $\begin{pmatrix}0\\a\end{pmatrix}$
The graph of $y = f(x)$ is transformed into the graph of: $y = f(x) + a$ by a translation of a units parallel to the y-axis or a translation by $\begin{pmatrix}0\\a\end{pmatrix}$
$y = f(x + a)$ by a translation of $-a$ units parallel to the x-axis or a translation by $\begin{pmatrix}-a\\0\end{pmatrix}$

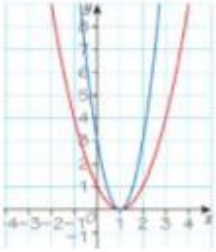




$y = f(-x)$ by a reflection in the y-axis

$y = -f(x)$ by a reflection in the x-axis

$y = af(x)$ by a stretch of scale factor a parallel to the y-axis



$y = f(ax)$ by a stretch of scale factor $\frac{1}{a}$ parallel to the x-axis