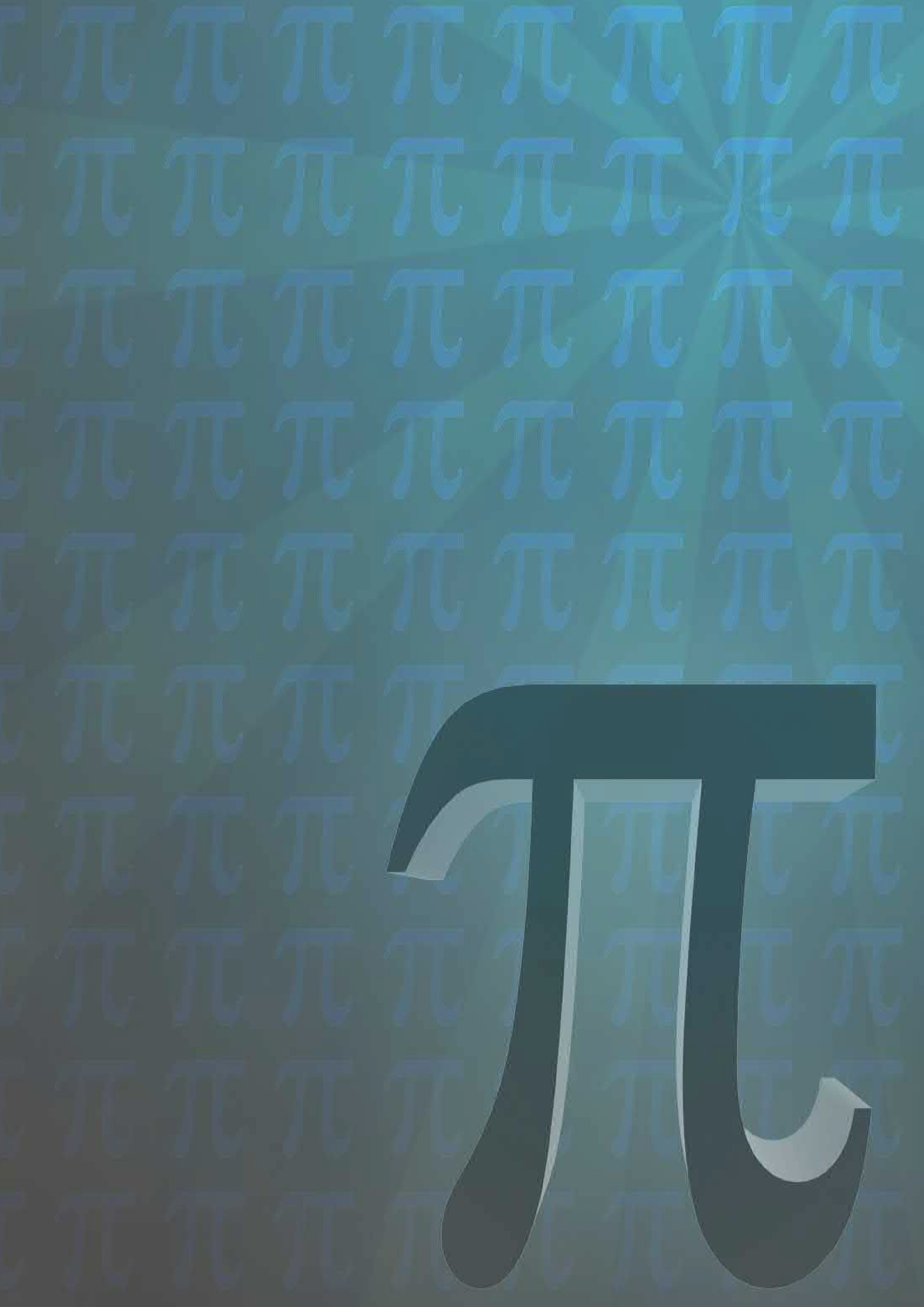
Exmouth Community College

KS4 Knowledge Organisers

MATHEMATICS

Units 1 - 10

HIGHER



Unit 2 Higher Algebra

Corbett Maths video links: V7 V13 V288

n^{th} term:

Example: For the following sequence, the first term (n = 1) is 2.
The 2^{nd} term (n = 2) is 5.

Positions (n numbers) →	1	2	3	4	5	6	. . . . n
TERMS →	2	5	8	11	14	17	. . . . .

+3 +3 +3 +3 +3

So we try rule: $n^{th \text{ term}} = 3n$. Testing the rule with n = 1 (1^{st} term) gives 3, and we know 1^{st} term should be 2, so we need an extra correction to rule of -1

So rule is: $\boxed{t_n = 3n - 1}$

67^{th} term is $t_{67} = 3 \times 67 - 1$
$= 200$

Simplifying expressions:

Gather together like terms, eg. **3e** *+ 2* **+ 4e** *– 8* = **7e** *+ 6*

Solving equations:

BALANCE METHOD:

You can use this on any equation, whether the unknown is on one side, or both

You can do whatever to like, so long as you do the *same* to both sides:

4f + 3 = 2f + 23

4f + 3	**=**	**2f + 23**	**[take 2f from each side]**
2f + 3	**=**	**23**	**[take 3 from each side]**
2f	**=**	**20**	**[divide both sides by 2]**
f	**=**	**10**	

If you want to get rid of something negative, ADD that same amount to both sides

Substitution:

Just like in sport, *substitution* means *swapping* one thing for another – but instead of a fresh player for a tired player, it's swapping a number for a letter.

If ⚽ = 5

then: ⚽ + 4 = 5 + 4 = 9

6 x ⚽ = 6 x 5 = 30

⚽ / 5 = 5 ÷ 5 = 1

Rather than drawing a football every time, they'd just use the letter "f"

When the expressions or formulae become a bit more complicated, it's *essential* that you follow the rules of BODMAS/BIDMAS:

e.g. If g = 10: 5 + 3g = 5 + 3 x 10
= 5 + 30
= 35

Classic exam question:

Bob works shifts in a café, where he get £6 a hour, plus a £5 travel bonus each day.

(a) Write a formula to describe his pay P for a day's shift of h hours: P = 6h + 5
(b) Use this formula to find his pay for a 7 hour shift: P = 6h + 5 = 6 x 7 + 5 = 42 + 5 = £47

Factorising

expanding brackets

3 (2t + 5) **6t + 15**

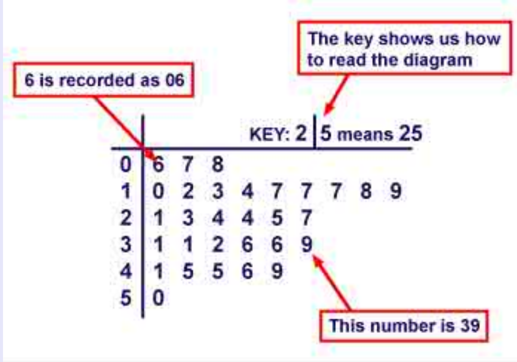
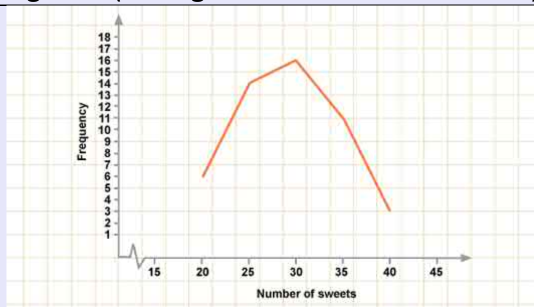
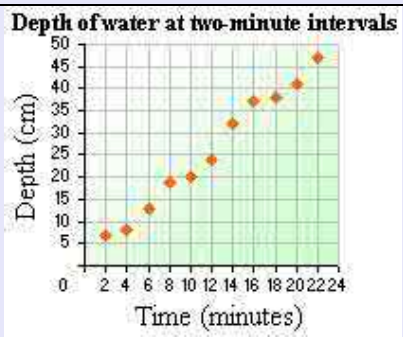
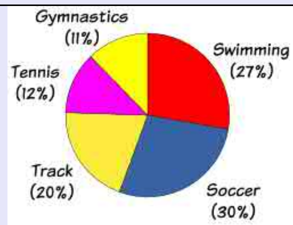
factorising

Expanding (2a+3)(4a+2)

	2a	+3
4a	$8a^2$	+12a
+2	+4a	+6

$8a^2 + 16a + 6$

Unit 3 Higher   Data

Mean: add up the numbers and divide by how many there are	*Median*: the 'middle' number. Order the numbers from smallest to largest and it's in the middle
Mode: the most commonly occurring number	*Range:* the difference between the largest and smallest numbers.
Stem & Leaf Diagram: a pictorial representation of grouped data The stem and leaf diagram is formed by splitting the numbers into two parts - in this case, tens (stem) and units (leaves). This information is given to us in the Key. It is usual for the numbers to be ordered.	
Frequency: the number of data points that fit into a category	*Correlation:* a mutual relationship or connection between two or more things. Can be positive (both go up at the same time) or negative (both go down at the same time).
Frequency polygon: a line graph that plots the frequency against the mid point of the group	
Modal Class: the class/group that has the highest frequency	*Medians in frequency tables:* if the total frequency is n then the median point lies in the class containing the $\frac{n+1}{2}$
	*Scatter graph:* used to represent and compare two sets of data. By looking at a scatter diagram, we can see whether there is any connection (correlation) between the two sets of data.
Line of best fit: A line of best fit is a straight line drawn through the center of a group of data points plotted on a scatter plot. Scatter plots depict the results of gathering data on two variables.	*Outlier:* a point which does not fit the overall pattern of a scatter graph.
Pie chart: a type of graph in which a circle is divided into sectors that each represent a proportion of the whole	

Unit 4 Higher (Fractions, Ratio, %)

Exmouth Community College Academy Trust

Corbett Maths video links: V271 V239 V234

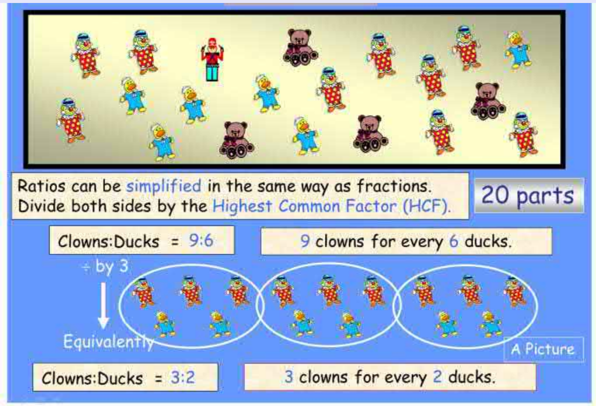
Fractions: Ratio, simplifying:

Reciprocal of n is $\frac{1}{n}$

To add and subtract mixed numbers, usually easier to convert them into *improper* (top-heavy) fractions, e.g.:

$$2\frac{1}{3}+5\frac{1}{4}=\frac{7}{3}+\frac{21}{4}$$

(then use Battenburg method)



Percentages of amounts

Calculator allowed?

Turn % into decimal (÷100) and "of" means "multiply".

e.g. 30% of £54 = 30 ÷100 x 54 = £16.20

e.g. 28% of £40 = 28 ÷100 x 40 = £11.20

Reverse percentages:

Use the logic of function machines, which can be run backwards.

You need to figure out the forwards multiplier first.

e.g. $30 dress reduced by 20%:

$30 → X 0.8 → $24

e.g. Sale price after 30% discount = £28

? → X 0.7 → £28

Original price £40 ← ÷ 0.8 ← £28

Calculator *not* allowed?

10% is your starting point.

10% means "a tenth of the amount"

(because 10% = 10/100 = 1/10)

You can work out all the other percentages you need by scaling up or down from 10%

e.g. 30% of £54?

10% = £5.40 (a tenth of 54 = 54/10)

20% = £10.80 (20% is double 10%)

30% = £16.20 (30% = 10% + 20%)

e.g. 28% of £40?

10% = £4

1% = 40p (divide 10% by 10)

2% = 80p (double 1%)

5% = £2 (half 10%)

20% = £8 (double 10%)

28% = these 4 added together, = £11.20

Battenburg: adding

1. Draw the battenburg grid.
2. Put the fractions on the side, (left to right, top to bottom).
3. Eat the top left corner (cross it out).
4. Do the multiplications.
5. "**ADD** the peanut" (the yellow ones below).
6. Peanut answer is numerator, the remaining number is denominator.
7. Simplify the fraction, if possible.

$$\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$$

	1	4
1	X	4
3	3	12

Divide top and bottom of fraction with the HCF that they share

Battenburg: subtracting

1. Draw the battenburg grid.
2. Put the fractions on the side, (left to right, top to bottom).
3. Eat the top left corner (cross it out).
4. Do the multiplications.
5. "**SUBTRACT** the peanut" (the yellow ones below).
6. Peanut answer is numerator, the remaining number is denominator.
7. Simplify the fraction, if possible.

$$\frac{1}{4}-\frac{1}{3}=\frac{1}{12}$$

	1	4
1	X	4
3	3	12

Divide top and bottom of fraction with the HCF that they share

Fractions, decimals, percentages conversion

Example:

Fraction → numerator ÷ denominator → **decimal** → X 100 → **percentage**

percentage → ÷ 100 → **decimal** → Write as fraction out of 10, 100, or 1000, and simplify → **Fraction**

Fraction 24/100 = 6/25 | decimal 0.24 | percentage 24%

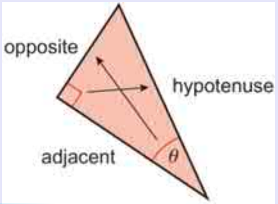
Some examples:

1/10 =	10/100 =	0.1 =	10%
1/5 =	20/100 =	0.2 =	20%
3/10 =	30/100 =	0.3 =	30%
9/20 =	45/100 =	0.45 =	45%

People often assume a % cannot be over 100, but it can (just like a fraction can be improper* and a decimal can be over 1)

*** top-heavy**

Unit 5 Higher Angles and Trigonometry



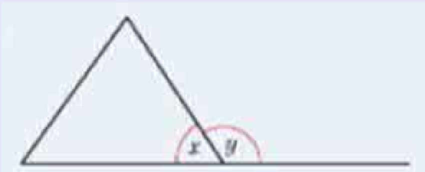
In a right-angled triangle, the longest side is called the hypotenuse and is opposite the right-angle.

The side opposite the angle θ is called the **opposite.**

The side that is next to angle θ is the **adjacent**.

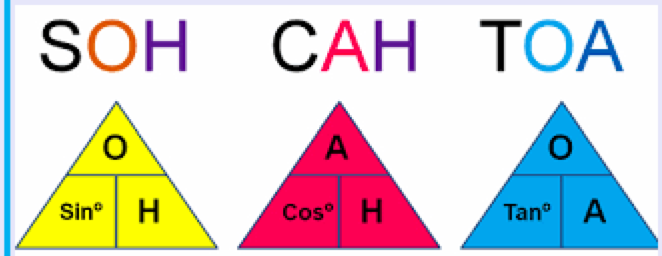
When one side of a triangle is extended at the vertex, it forms an **exterior** angle.
x is the **interior** angle.
y is the **exterior** angle.

$$x + y = 180^{0}$$



The sum of the interior angles of a polygon with n sides = (n-2) x 180^{0}

The sum of the **exterior** angles of a polygon is always 360^{0}

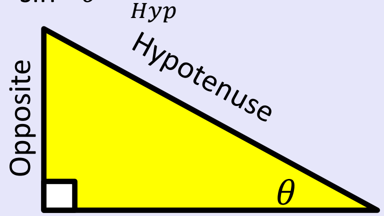


Sine Ratio

Opp = sinθ x *Hyp*

Hyp = $\frac{Opp}{\sin\theta}$

$\sin^{-1}\theta = \frac{Opp}{Hyp}$

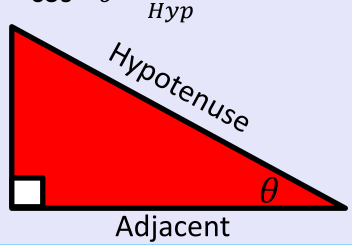


Cosine Ratio

Adj = cosθ x *Hyp*

Hyp = $\frac{Adj}{\cos\theta}$

$\cos^{-1}\theta = \frac{Adj}{Hyp}$

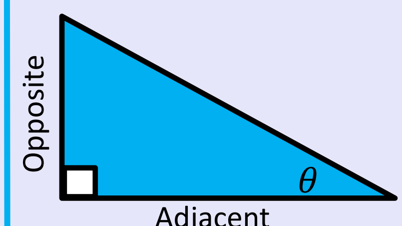


Tangent Ratio

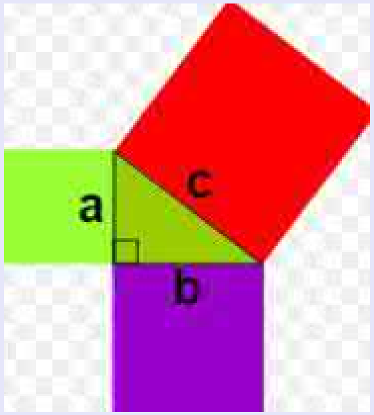
Opp = tanθ x *Adj*

Adj = $\frac{Opp}{\tan\theta}$

$\tan^{-1}\theta = \frac{Opp}{Adj}$



To get $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ press shift on the calculator and then the corresponding ratio.



Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

To find **hypotenuse**:
Square side a
Square side b
Add together
Square root

To find shorter side:
Square side c
Square side a or b
Subtract a or b from c
Square root

V257

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	

The exact **sine**, **cosine** and **tangent** of some angles are in this table.

V329
V330
V331

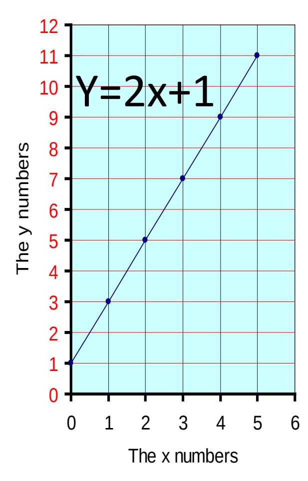
Unit 6 Higher Graphs – Links: V191 V171 V197 V196

Linear Equations

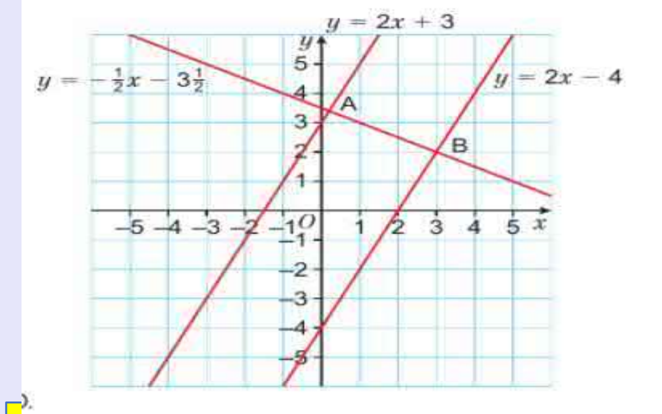
Y = m*x* + c

where m is the gradient

C is where the graph crosses the y axis



Parallel lines have same gradient



Perpendicular lines have gradients that multiply to give -1

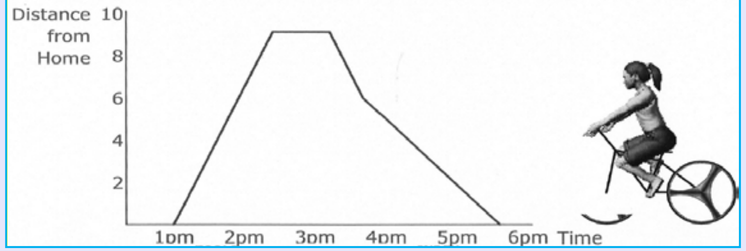
When a graph has gradient m, the perpendicular line to that will have gradient - $\frac{1}{m}$

A distance – time graph represents a journey

The gradient is the speed

Try to draw a graph which reflects this cyclist's journey

At 1pm she starts off on a journey of 9 miles. She gets there by 2:30pm
She stays there for 45 minutes.
Then she travels for 3 miles in direction of home which takes 30 minutes.
The cyclist then gets a puncture and takes 2hrs to do the last 6 miles home.



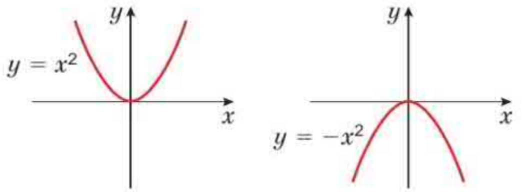
Velocity- time graph

Straight line – means constant acceleration

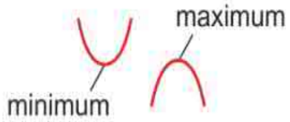
Direct proportion is shown by a straight line graph through the origin

The equation of a circle with centre (0,0) and radius r is $x^2 + y^2 = r^2$

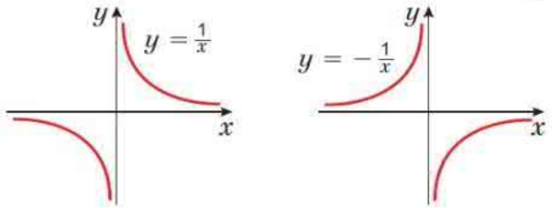
A **quadratic equation** contains a term in x^2 but no higher power of x.
The graph of a quadratic equation is a curved shape called a **parabola**.



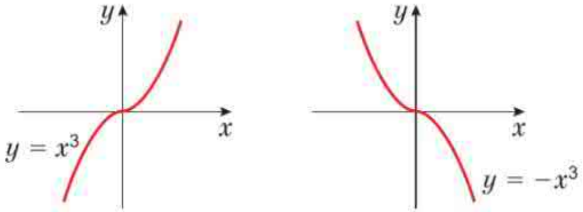
A quadratic graph has either a **minimum point** or a **maximum point** where the graph turns.



Reciprocal functions are in the form $\frac{k}{x}$ where k is a number.



A **cubic function** contains a term in x^3 but no higher power of x.
It can also have terms in x^2 and x and number terms.



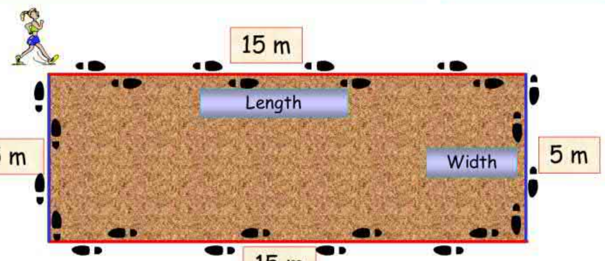
Knowledge Organiser: Unit 7 Higher (Area and Volume)

Corbett Maths video links: V312 V377 V358

Perimeter

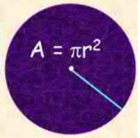
The perimeter of a shape is the distance around the outside.

Rectangles

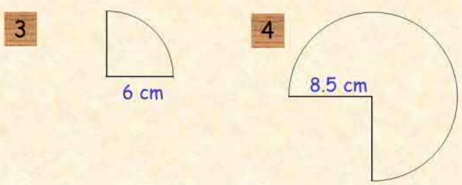


Perimeter = 15 m + 5 m + 15 m + 5 m = 40 m

The Area of a Circle



Find the area of the $\frac{1}{4}$ and $\frac{3}{4}$ circles.



$A = \frac{1}{4}\pi r^2$
$= \frac{1}{4} \times \pi \times 6^2$
$=$ 28.3 cm² (1 dp)

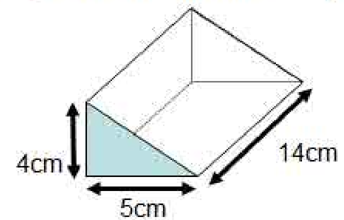
$A = \frac{3}{4}\pi r^2$
$= \frac{3}{4} \times \pi \times 8.5^2$
$=$ 170.2 cm² (1 dp)

Volume

VOLUME is how many cubic units fit **inside** a shape.

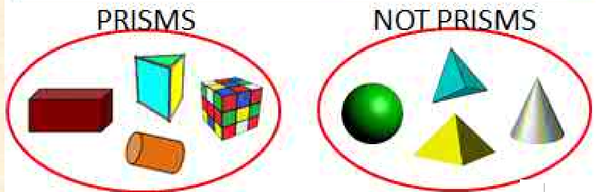
For a prism* Volume = Area x length

*a shape that is the same all the way along its length



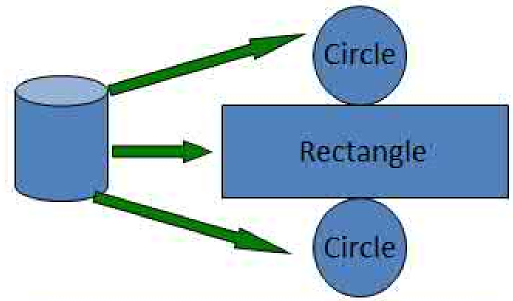
$A = \frac{1}{2} \times 4 \times 5 = 10\text{cm}^2$ $V = A \times L = 10 \times 14 = 140\text{cm}^3$

So, always start by working out the **area** on front of the shape – this has to be the same all the way along the length (i.e. it has to be a prism).



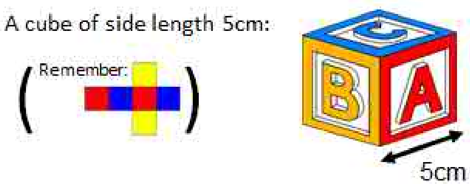
Surface Area

SURFACE AREA is how many square units fit onto the **outside** of a shape.



It's helpful to think of the net of the shape: the surface area is just the area of all the bits of the net added together.

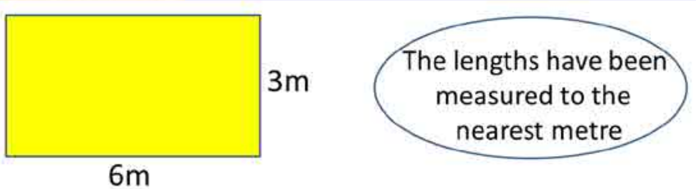
e.g. A cube of side length 5cm:



Area of one face = 5 x 5 = 25cm²

Total surface area = 25 x 6 = 150cm²

Error bounds:



What the minimum and maximum values that the base and height could be?

5.5 ≤ base < 6.5m 2.5 ≤ height < 3.5m

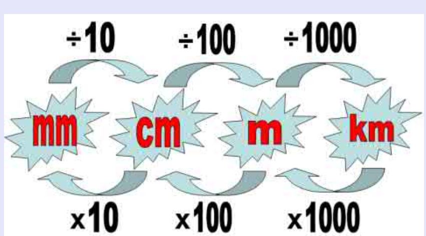
What the minimum and maximum values that the *perimeter* could be?

16m ≤ perimeter < 20m

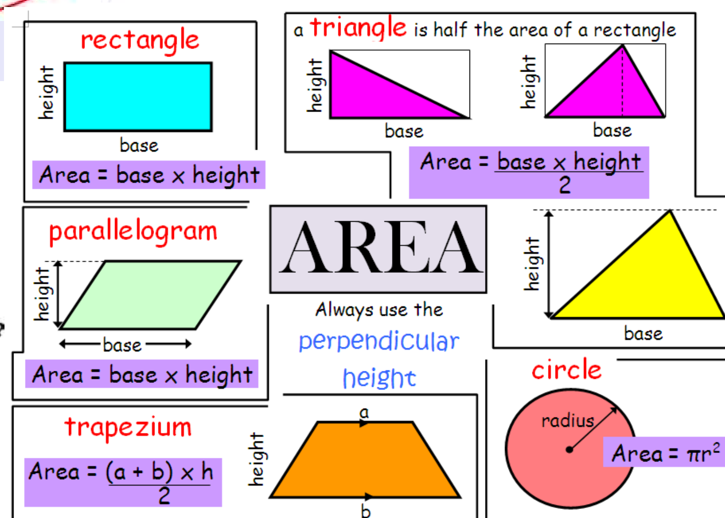
What the minimum and maximum values that the *area* could be?

13.75m² ≤ area < 22.75m²

Metric conversions:



AREA

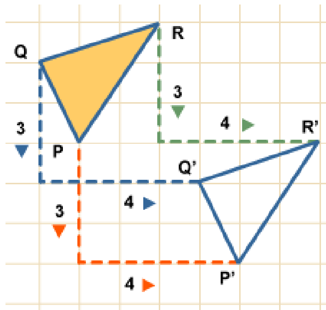


KS4 Knowledge Organiser

Higher Tier Unit 8: Transformations & Constructions

Translation: V325

To translate means to move a shape. The shape does not change size or orientation.



Column Vector:

In a column vector, the top number moves left (-) or right (+) and the bottom number moves up (+) or down (-)

$\begin{pmatrix}2\\3\end{pmatrix}$ means '2 right, 3 up'

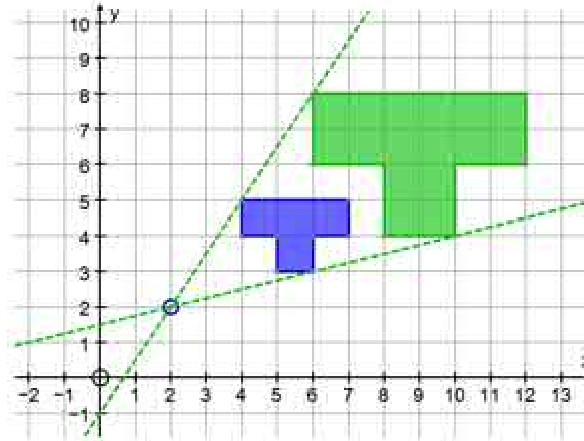
$\begin{pmatrix}-1\\-5\end{pmatrix}$ means '1 left, 5 down'

Enlargement:

The shape will get **bigger** or **smaller**. Multiply each side by the **scale factor**.

Scale Factor = 3 means '3 times larger = multiply by 3'

Scale Factor = ½ means 'half the size = divide by 2'

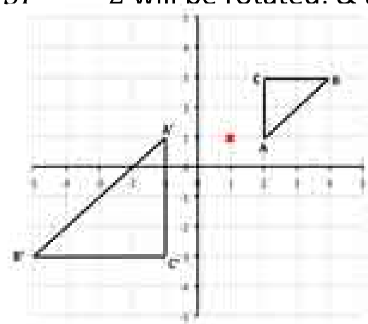


V104
V105
V106

V107 V108

Negative Scale Factor Enlargements will look like they have been rotated.

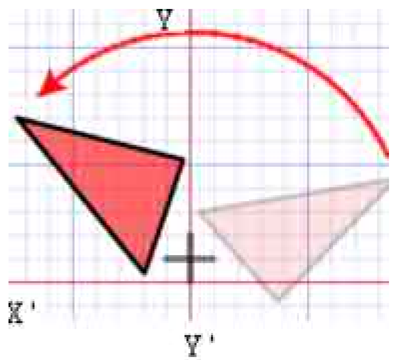
$SF = -2$ will be rotated. & also twice as big. Enlarge ABC by scale factor -2, centre (1,1)



Rotation: V275

The size does not change, but the shape is turned around a point. (Use tracing paper).

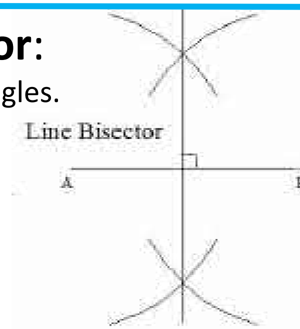
Rotate the triangle 90° anti-clockwise about (0,1).



Perpendicular Bisector:

Cuts a line in half and at right angles.

V78



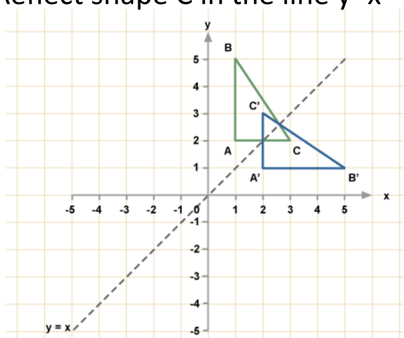
Reflection:

The size does not change, but the shape is '**flipped**' like in a **mirror**.

Line **x=?** is a **vertical line.**
Line **y=?** is a **horizontal line.**
Line **y=x** is a **diagonal line.**

V272 V273 V274

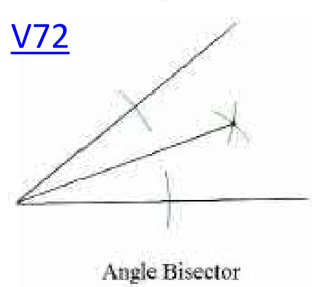
Reflect shape C in the line y=x



Angle Bisector:

Cuts the angle in half.

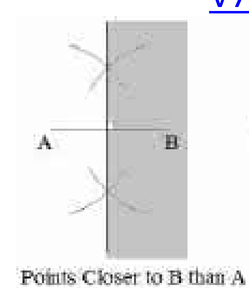
V72

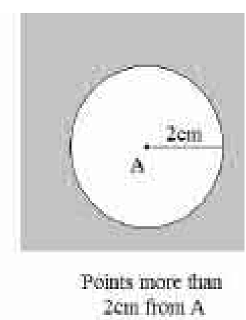


Loci:

A locus is a path of points that follow a rule.

V75 V76 V77





KS4 Knowledge Organiser

Higher Tier Unit 9: Equations & Inequalities

Exmouth Community College

Quadratic: V325

A quadratic expression is of the form **$ax^2 + bx + c$**
where a, b and c are numbers, **$a \neq 0$**
Examples of quadratic expressions: **x^2 or $8x^2 - 3x + 7$**

Factorising Quadratics: V118 V119

When a quadratic expression is in the form $x^2 + bx + c$ find the 2 numbers that add to give b & multiply to give c.
e.g. $x^2 + 7x + 10 = (x+5)(x+2)$
(because 5 and 2 add to give 7 and multiply to give 10)

Difference of Two Squares V120

An expression of the form a^2-b^2 can be factorised to give $(a+b)(a-b)$.
e.g. $x^2 – 25 = (x+5)(x-5)$ or $16x^2 – 81 = (4x+9)(4x-9)$

Solving Quadratics ($ax^2 = b$)

Isolate the x^2 term and square root both sides.
e.g. $2x^2 = 98$
$x^2 = 49$
$x = \pm 7$

Remember there will be a positive and a negative solution.

Solving Quadratics ($ax^2 + bx = 0$) V266

Factorise and then **solve = 0**

e.g. $x^2 - 3x = 0$
$x(x-3) = 0$
$x = 0$ or $x = 3$

e.g. Solve $x^2 + 3x – 10 = 0$
Factorise: $(x+5)(x-2) = 0$
$x=-5$ or $x=2$

Simultaneous Equations:

A set of two or more equations, each involving two or more variables (letters).
The solutions to simultaneous equations satisfy both/all of the equations.
V295 V296 V297

e.g. $2x + y = 7$
$3x – y = 8$
$x=3, y=1$

Factorising Quadratics when a ≠ 1 V266

When a quadratic is in the form **$ax^2 + bx + c$**
1. Multiply a by c = ac
2. Find two numbers that add to give b and multiply to give ac.
3. Re-write the quadratic, replacing bx with the two numbers you found.
4. Factorise in pairs – you should get the same bracket twice
5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.

Completing the Square V267a V371

A quadratic in the form **$ax^2 + bx + c$** can be written in the form **$(x + p)^2 + q$**
1. Write a set of brackets with x in and half the value of b.
2. Square the bracket.
3. Subtract $(b/2)^2$ and add c.
4. Simplify the expression.

Solving Quadratics using the Quadratic Formula: V267

A quadratic in the form **$ax^2 + bx + c$** can be solved using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

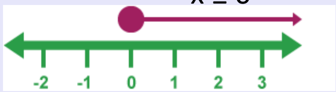
Use the formula if the quadratic does not factorise easily.

Inequality symbols: V176 V177 V178 V179

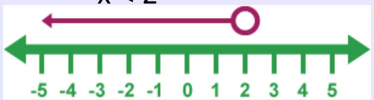
$x>2$ means x is **greater than** 2
$x<3$ means x is **less than** 3
$x\geq 1$ means x is **greater than or equal to** 1
$x\leq 6$ means x is **less than or equal to** 6

Inequalities can be shown on a number line.
Open circles are used for numbers that are **less than or greater than** (< or >)
Closed circles are used for numbers that are **less than or equal or greater than or equal** (≤ or ≥)

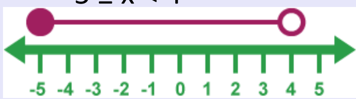
$x \geq 0$



$x < 2$



$-5 \leq x < 4$



Unit 10 Higher (Probability)

Corbett Maths video links: V244 V250 V247

TECHNICAL LANGUAGE:

P("something") means *probability of "something" happening*

"Mutually exclusive" means that if one thing happens, the other cannot. E.g. being alive and dead are mutually exclusive states!

"Bias" = unfairness. It would be biased to roll a die that has 2 sixes on it and no zeroes in a normal dice game.

If outcomes A and B are mutually exclusive, P(A) + P(B) = 1 or 1-P(A) = P(B)

E.g. If there is no draw allowed, and P(win) = 0.7, B(lose) must be 0.3

Sometimes bias is difficult to spot in experiments. If you flip a coin 100 times, you expect 50 heads and 50 tails, but does that mean your coin is biased if you get 60:40? What about 90:10?? What about 99:1????

On fair dice, opposite faces *should* add up to 7.

COMBINING PROBABILITIES:

If you want to find the probability of 2 things happening, MULTIPLY the individual probabilities.

One of the reasons why fractions are convenient for probability is That they are so easy to multiply; ½ x ⅝ = 5/16

Multiply numerators, multiply denominators

Remember to simplify whenever possible

Example:

P(win 🔍) = 2/5   P(win ✓) = 3/10   P(win both) = 2/5 x 3/10 = 6/50 = 3/25

The LANGUAGE of probability:

P("something") means *probability of "something" happening*

Eg. When tossing a coin P(heads) = 0.5 or ½

P(tails) = 0.5 or ½

P(heads or tails) = 1 (certain)

P(coin flying off into outer space) = 0 (impossible)

It's often easiest to write probabilities as **fractions***, especially if you want to combine probabilities in tree diagrams...

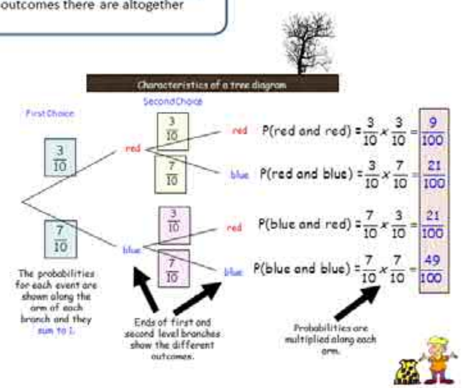
$$* \ \frac{\text{how many ways it can happen}}{\text{How many outcomes there are altogether}}$$

Sample Space Diagrams:
Often used to find all the possible combinations of 2 events being combined:

Roll a die

Roll a die	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

If we're adding, The value in the (6,6) box of the SSD would be 12



You can use two-way tables to help solve probability problems:

	France	Holland	Elsewhere	Total
June	6	18	5	29
July	10	19	2	31
August	15	15	10	40
Total	31	52	17	100

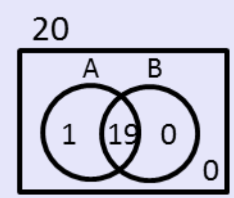
What is the probability that a person selected at random:

1. Went to Holland on holiday?	52/100
2. Went on holiday in July?	31/100
3. Went to France in August?	15/100
4. Did not visit either France or Holland?	17/100
5. Went on holiday in June?	29/100

Exmouth Community College Academy Trust

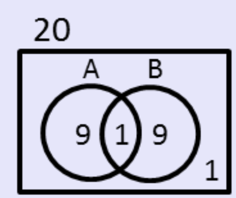
VENN DIAGRAMS

$$P(A \cap B) = \frac{19}{20}$$



20 people chose A, and 19 chose B.

$$P(A \cup B) = \frac{19}{20}$$



1 person opts out of choosing either A or B.

HCF and LCM V219 V218

(Highest Common Factor and Lowest Common Multiple)

HCF - this is largest number that divides exactly into 2 or more numbers. E.g. HCF or 12 and 20 = 4
LCM - this is the smallest number that is in the times table of 2 or more numbers. E.g. LCM of 12 and 20 = 60

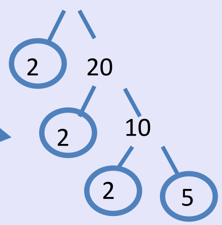
Product of Prime Factors V219

This is finding all the prime numbers that would multiply to give our number. It is often shown using a factor tree ('tree thingy'). V223

Eg. 40 as a product of prime factors

We are looking for the prime numbers that go into our number

40 = 2 x 2 x 2 x 5



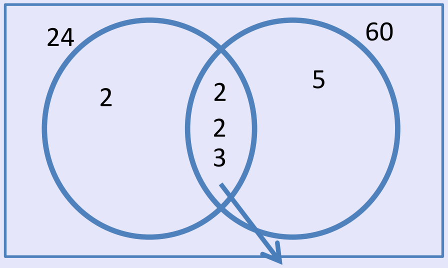
We can then multiply our prime numbers together to get our original number

Using product of prime factors to find our HCF and LCM
Example: Find the HCF and LCM of 24 and 60

Step 1:
24 = 2 x 2 x 2 x 2
60 = 2 x 2 x 3 x 5

Write each number as a product of prime factors

Step 2: Draw a Venn Diagram V224



Place you prime factors into your Venn diagram

The HCF of 24 and 60 = 2 x 2 x 3 = 12

Multiply the common prime factors

The LCM of 24 and 60 = 2 x 2 x 2 x 3 x 5 = 120

Multiply all the prime factors

Unit 1 Higher Number

Exmouth Community College

Laws of Indices V17

Index/power

3^3

base

When multiplying indices with the same base, you ADD the indices

Multiplying Indices

$y^3 \times y^3 = y \times y \times y \times y \times y \times y = y^6$

Dividing Indices

$y^6 \div y^4 = \dfrac{y \times y \times y \times y \times y \times y}{y \times y \times y \times y} = y^2$

When dividing indices with the same base, you SUBTRACT the indices

Power to another power (brackets)

$(y^3)^2 = (y \times y \times y)^2$
$= y \times y \times y \times y \times y \times y = y^6$

With brackets just MULTIPLY your indices

Zero Indices

$y^0 = 1$

Anything to the power of 0 always equals 1

Negative Indices V175

$y^{-1} = \dfrac{1}{y}$

$y^{-2} = \dfrac{1}{y^2}$

The negative sign means 'one over' the base number

e.g. $5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

Fractional Indices

$y^{\frac{2}{3}} = (\sqrt[3]{y})^2$

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 4$

The denominator of the fractional power becomes a root and the numerator becomes a power

V173

Standard Form

V300 V301 V302 V303

A number is in standard form when it is in the form A x 10^n, $where\ 1 \le A < 10$.

For example, 63000 = 6.3 x 10^4. This is in standard form because 6.3 is between 1 and 10. 63 x10^4 is not in standard form as 63 is not between 1 and 10.

Standard form is used so very large or very small numbers can be written out easily.

Examples
45 000 000 000 = 4.5 x 10^{10}
0.0000000000091 = 9.1 x 10^{-12}

Surds

A surd is a number written exactly using square or cube roots.
For example $\sqrt{3}$ and $\sqrt{5}$ are surds. $\sqrt{4}$ and $\sqrt[3]{27}$ are not surds, because $\sqrt{4}$ = 2 and $\sqrt[3]{27}$ = 3.

Multiplying Surds

$\sqrt{m} \times \sqrt{n} = \sqrt{m \times n} = \sqrt{mn}$
E.g. $\sqrt{3} \times \sqrt{2} = \sqrt{3 \times 2} = \sqrt{6}$

Dividing Surds

$\sqrt{m} \div \sqrt{n} = \sqrt{\dfrac{m}{n}}$

E.g. $\sqrt{12} \div \sqrt{3} = \sqrt{\dfrac{12}{3}} = \sqrt{4} = 2$

V305 V306 V307 V308

Unit 11 Multiplicative Reasoning V236 V384 V385 V254 V255

In **compound interest** the interest earned each year is added to money in the account and earns interest the next year.
Most interest rates are compound interest rates.

You can calculate an amount after n years' compound interest using the formula

$$\text{amount} = \text{initial amount} \times \left(\frac{100 + \text{interest rate}}{100}\right)^n$$

If y is directly proportional to x, $y \propto x$ and $y = kx$, where k is a number, called the **constant of proportionality**.

Where k is the constant of proportionality:

- if y is proportional to the square of x then $y \propto x^2$ and $y = kx^2$
- if y is proportional to the cube of x then $y \propto x^3$ and $y = kx^3$
- if y is proportional to the square root of x then $y \propto \sqrt{x}$ and $y = k\sqrt{x}$

These are three kinematics formulae:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

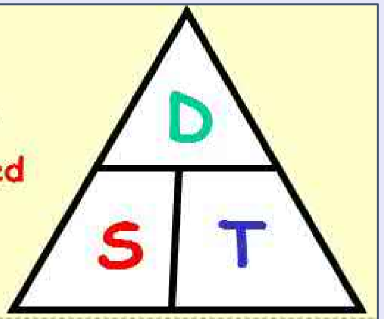
$v^2 = u^2 + 2as$

where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken

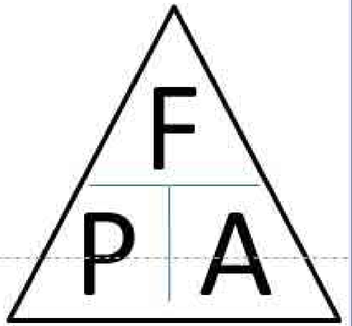
Multiplicative means involving multiplication or division

Key Words
Velocity
Acceleration
Force
Pressure

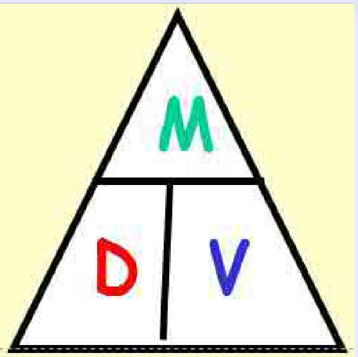
Distance = Time X Speed
Speed = Distance ÷ Time
Time = Distance ÷ Speed



Force = Pressure x Area
Pressure = Force ÷ Area
Area = Force ÷Pressure



Mass = Density x Volume
Density= Mass ÷ Volume
Volume = Density ÷ Mass

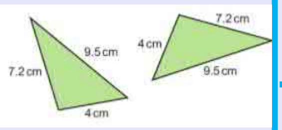


Unit 12 Higher Similarity and Congruence

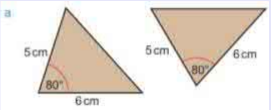
Congruent Triangles

Are exactly the same size and shape. Triangles are congruent when one of these conditions are true:

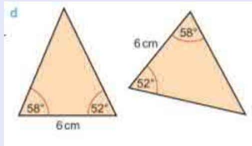
- SSS – all three sides are equal.



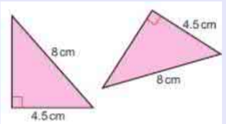
- SAS – two sides and included angle are equal.



- AAS – two angles and corresponding side are equal.



- RHS – right angle, hypotenuse and another side are equal.



V67

You need to prove it by using one of the above reasons.

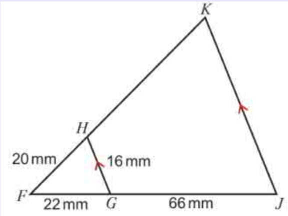
Similarity

V291

Shapes are similar when one shape is an enlargement of each other. Corresponding sides are in the same ratio. Corresponding angles are equal. When comparing two similar shapes, a scale factor can be found. This scale factor helps to find missing sides of the shape.

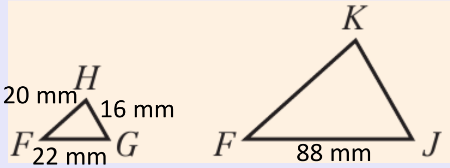
Similar Triangles

Prove FGH and FJK are similar.



Angle F occurs in both triangles. Therefore the same.

Draw the triangles separately.



FGH = FJK as corresponding angles.

FHG = FKJ as corresponding angles.

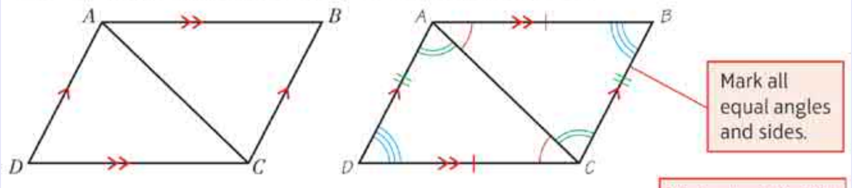
Therefore all angles are equal so triangle is similar.

Congruence is used to solve problems and prove that shapes are the same.

To prove it: write a series of logical statements. Each statement needs must be supported by a mathematical reason.

Proving Geometric Congruence

V66


ABCD is a parallelogram. Prove triangle ABC is congruent to ADC.

Mark all equal angles and sides.

Length AB = length CD because opposite sides in a parallelogram are equal. — State why AB = CD

Length BC = length AD because opposite sides in a parallelogram are equal. — State why BC = AD

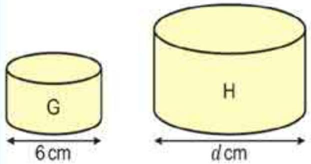
Length AC is common to both triangles.

So triangle ABC is congruent to triangle ADC (SSS). — State the condition used to prove congruence.

Similarity in 3D shapes

If a shape is enlarged by a linear scale factor of *k*, the area of the shape is enlarged by scale factor of k^2.

If a shape is enlarged by a linear scale factor of *k*, the volume of the shape is enlarged by scale factor of k^3.


Cylinders G and H are similar.
The diameter of G is 6 cm.
The volume of G is 108 cm³. The volume of H is 256 cm³
Work out the diameter *d* of cylinder H.

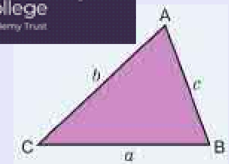
Volume scale factor = $\frac{\text{large}}{\text{small}} = \frac{256}{108} = \frac{64}{27} = k^3$

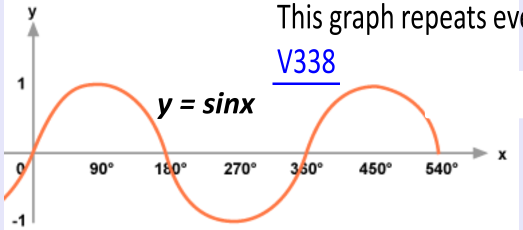
$k = \sqrt[3]{\frac{64}{27}} = \frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \frac{4}{3}$

$d =$

V293a
V293b

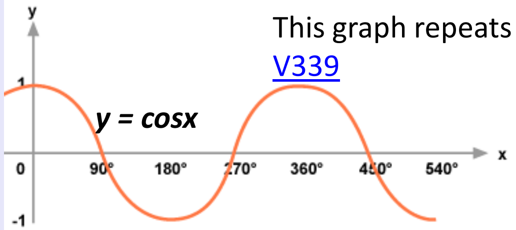
Unit 13 Higher More Trigonometry





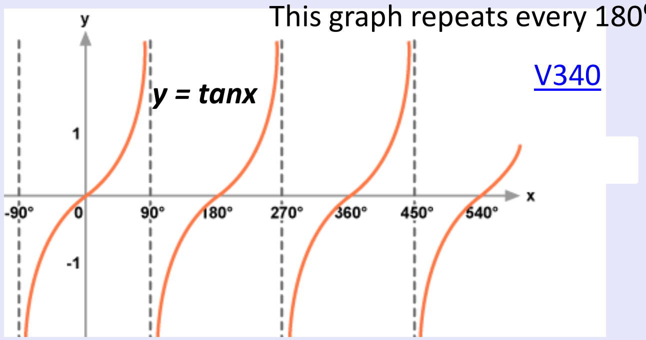
This graph repeats every 360°.

V338



This graph repeats every 360°.

V339



This graph repeats every 180°.

V340

Area of a triangle

$$\frac{1}{2}absinC$$

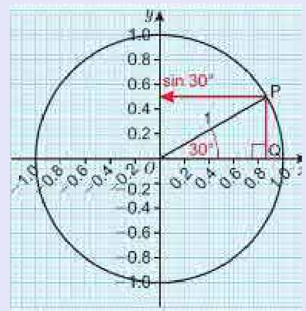
To be used when you can't use: $\frac{1}{2}$ base x height

Transforming trigonometric graphs

y= f(x) is a function where x is the input. The output is *y* or *f(x)*.

- *y= -f(x)* is a reflection in the *x*-axis.
- *y = f(-x)* is a reflection in the *y*-axis.
- *y=-f(-x)* is a reflection in the *y* and *x* axis. It is equivalent to a rotation of 180° about the origin.
- *y = f(x + a)* is a translation by $\binom{-a}{0}$ V323
- *y=af(x)* is a vertical stretch by scale factor *a*, parallel to the *y*-axis.
- *Y=f(ax)* is a horizontal stretch by the scale factor $\frac{1}{a}$

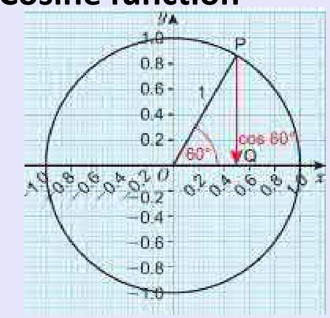
Sine function



The diagram shows a circle with radius 1 and centre (0,0). The length of PQ gives the sine of the angle.

sin30° = $\frac{PQ}{1}$ = PQ = 0.5

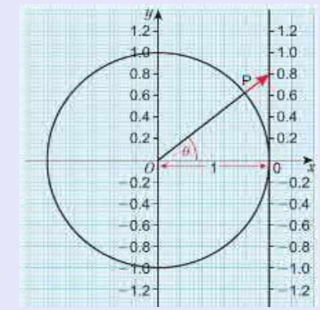
Cosine function



The diagram shows a circle with radius 1 and centre (0,0). The length of OQ gives the cosine of the angle.

cos60° = $\frac{OQ}{1}$ = OQ = 0.5

Tangent function



The diagram shows a circle with radius 1 and centre (0,0). Extend OP to hit the tangent. This gives a value for tanθ.

tanθ = $\frac{0.8}{1}$ = 0.8

The sine rule

$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ To find a side.

V333

$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$ To find an angle.

Can be used in any triangle. You need to know one angle and the opposite side. Then:

- If you know another angle, you can calculate its opposite side.
- If you know another side, you can calculate the opposite angle.

The cosine rule

V335 V336

$a^2 = b^2 + c^2 - 2bccosA$ To find a side.

cosA = $\frac{b^2 + c^2 - a^2}{2bc}$ To find an angle.

Can be used in any triangle. Use it to find:

- The length of a side if you know two sides and the included angle
- An unknown angle if you know all three sides.

3D Pythagoras

V259

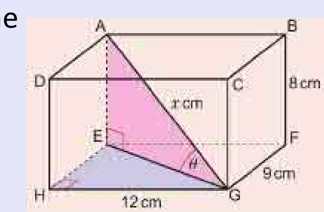
A plane is a flat surface.

EFGH is a horizontal plane. AEG is a vertical plane.

AG is the diagonal named x. To calculate the value of x, you need to find the value of EG using Pythagoras' Theorem.

EG: $\sqrt{12^2 + 9^2}$ = 15cm

x: $\sqrt{15^2 + 8^2}$ = 17cm



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Unit 14 Higher Further Statistics V149 V159 V154 V281

Key Words

Population
Census
Sample
Bias
Random Sample
Strata

Histogram

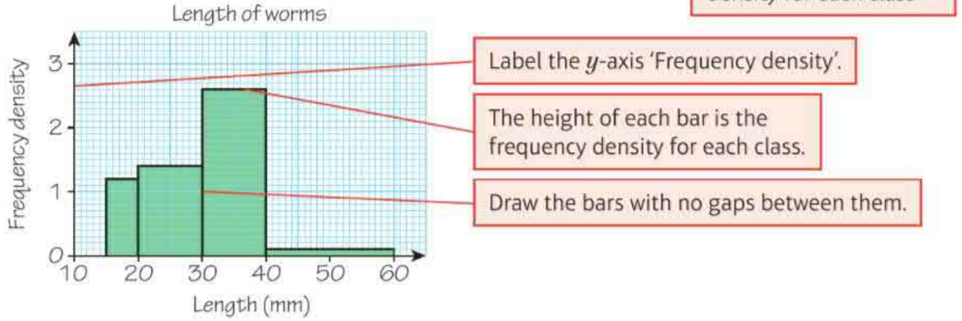
The lengths of 48 worms are recorded in this table.

Length, x (mm)	$15 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 60$
Frequency	6	14	26	2

Draw a histogram to display this data.

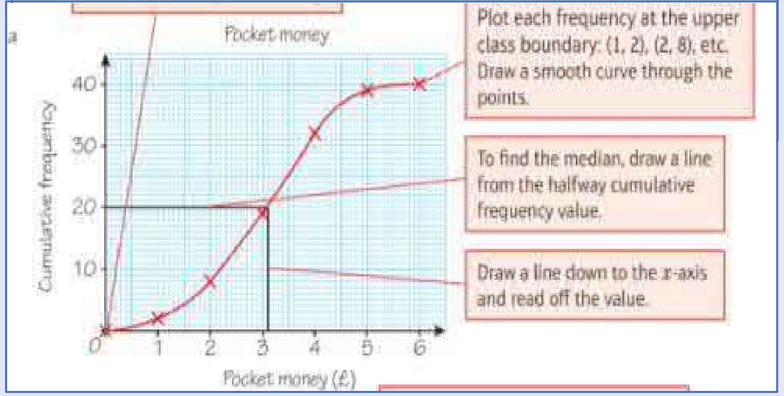
$6 \div 5 = 1.2,\ 14 \div 10 = 1.4,\ 26 \div 10 = 2.6,\ 2 \div 20 = 0.1$

Work out the frequency density for each class



Cumulative Frequency

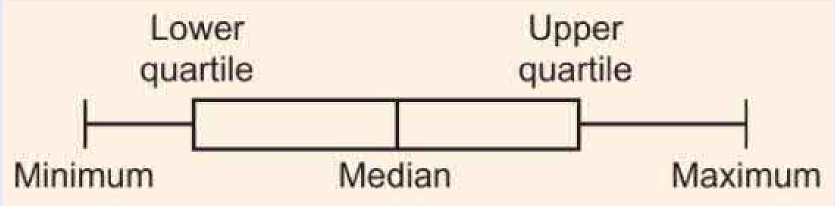
Pocket money, x (£)	Cumulative frequency
$0 < x \leq 1$	2
$0 < x \leq 2$	8
$0 < x \leq 3$	19
$0 < x \leq 4$	32
$0 < x \leq 5$	39
$0 < x \leq 6$	40



Stratified Sample

$$\frac{Sample}{Population} \text{ X Stratum Size}$$

Box Plot



Range is Max – Min

Catch and Release

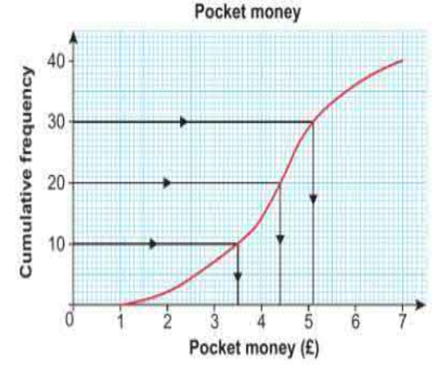
$$\frac{n}{N} = \frac{m}{M}$$

$$\text{So } N = \frac{n}{m} \times M$$

Always Compare a measure of 'spread' and a value

The **median** and **quartiles** can be estimated from the cumulative frequency diagram. For a set of n data values

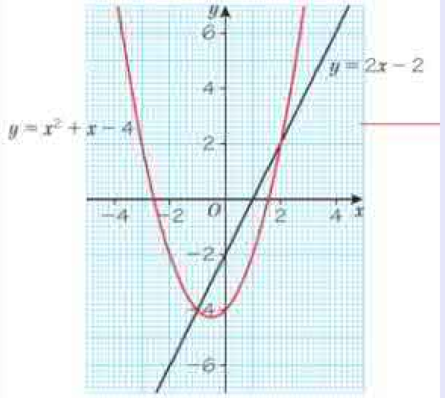
- the estimate for the **median** is the $\frac{n}{2}$th value
- the estimate for the **lower quartile** (LQ) is the $\frac{n}{4}$th value
- the estimate for the **upper quartile** (UQ) is the $\frac{3n}{4}$th value
- the **interquartile range** (IQR) = UQ – LQ



Quadratic Graphs

V180 V181 V276c VCubic

Solving Simultaneous Equations

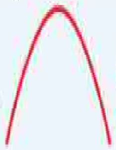


The solutions are
$x = 2, y = 2$ and $x = -1, y = -4$

The lowest or highest point of the parabola, where the graph turns, is called the **turning point**.

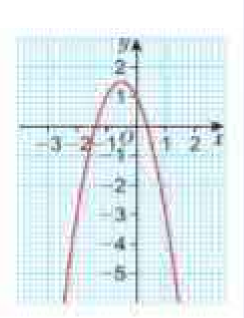
The turning point is either a minimum or maximum point.

The x-values where the graph intersects the x-axis are the solutions, or **roots**, of the equation $y = 0$.

 is a minimum,  is a maximum

To sketch a quadratic function

- Calculate the solutions to the equation '$y = 0$' (points of intersection with the x-axis).
- Calculate the point at which the graph crosses the y-axis.
- Find the coordinates of the turning point and whether it is a maximum or a minimum.

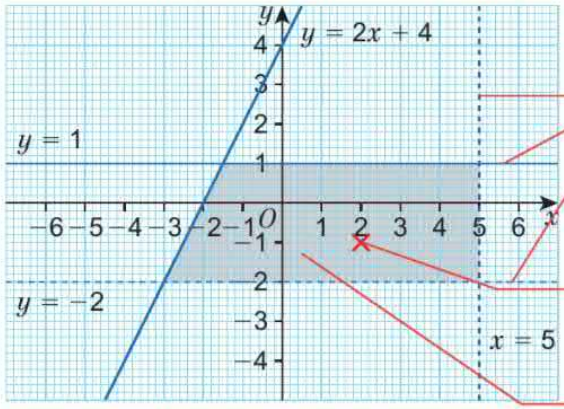


When a quadratic is written in completed square form $y = a(x + b)^2 + c$ the coordinate of the turning point is $(-b, c)$

The quadratic equation $ax^2 + bx + c = 0$ is said to have no real roots if its graph does not cross the x-axis. If its graph just touches the x-axis, the equation has one repeated root.

Inequality Graphs

On a coordinate grid, shade the region that satisfies the inequalities
$x < 5$, $y \leqslant 2x + 4$, $y \leqslant 1$ and $y > -2$



Draw dotted lines $x = 5$ and $y = -2$
Draw solid lines $y = 2x + 4$, $y = 1$

Test a point. For $(2, -1)$
$y \leqslant 1$ and $y > -2$: the y-coordinate is -1
$x < 5$: the x-coordinate is 2
$2x + 4 = 8$: y-coordinate $\leqslant 8$

Shade the required region.

Cubic Graphs

A **cubic** function is one whose highest power of x is x^3.
It is written in the form $y = ax^3 + bx^2 + cx + d$

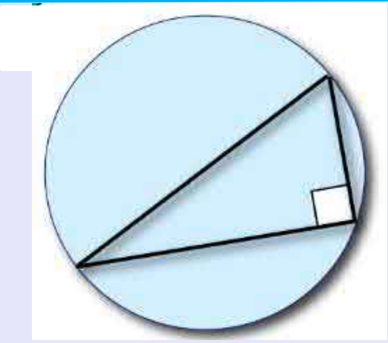
When $a > 0$ the function looks like  or 

When $a < 0$ the function looks like  or 

The graph intersects the y-axis at the point $y = d$
The graph's roots can be found by finding the values of x for which $y = 0$.

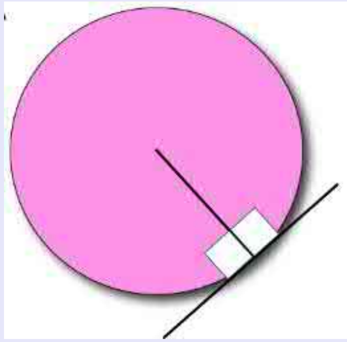
Unit 16 Higher Circle Theorems

V64 V65



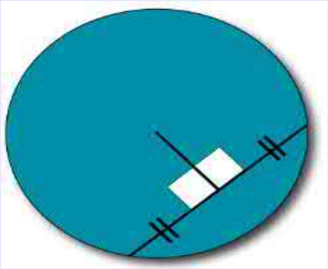
The angle in a semicircle is a right angle.

V65a



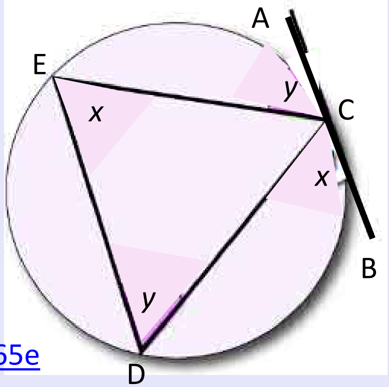
The angle between a **tangent** and **radius** is a right angle.

V65f



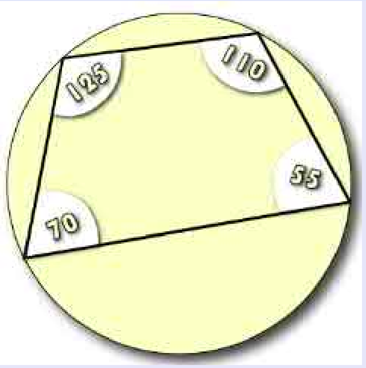
A **chord** is a straight line connecting two points on a circle.

The **perpendicular** from the centre of the circle to a chord **bisects** the chord and the line drawn from the centre of the circle to the **midpoint** of a chord is at right angles to the chord.



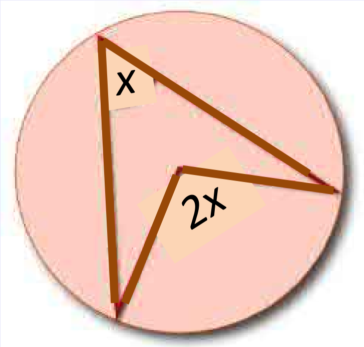
V65e

AB is a tangent to the circle. CD, DE and CE are **chords**. Angle ECA is the angle between the **tangent** and the chord in one segment. The other **segment** has angle CDE. This is the **alternate segment.** The angle between the chord and tangent is equal to the angle in the alternate segment.



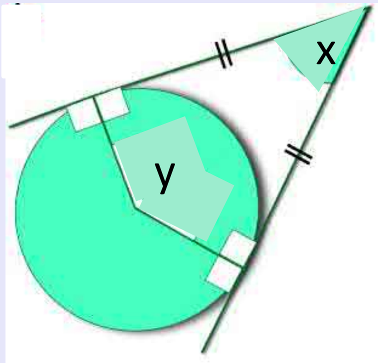
A cyclic quadrilateral with all four vertices on the circumference of the circle. Opposite angles add up to 180^o.

V65d



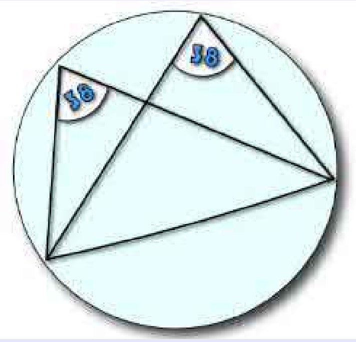
The angle at the centre of a circle is twice the angle at the circumference when both are subtended by the same arc.

V65b



Tangents drawn from a point outside the circle are equal in length.

$x + y = 180$



Angles in the same segment and standing on the same chord are always equal.

V65c

Unit 17 Higher More Algebra V8 V21 V22 V23 V24 V365 V370

You can change the subject of a formula by isolating the terms involving the new subject.

When the letter to be made the subject appears twice in the formula you will need to factorise.

You may need to factorise before simplifying an algebraic fraction:

- Factorise the numerator and denominator.
- Divide the numerator and denominator by any common factors.

You may need to factorise the numerator and/or denominator before you multiply or divide algebraic fractions.

To add or subtract algebraic fractions, write each fraction as an equivalent fraction with a common denominator.

To find the lowest common denominator of algebraic fractions, you may need to factorise the denominators first.

To rationalise the fraction $\frac{1}{a \pm \sqrt{b}}$, multiply by $\frac{a \mp \sqrt{b}}{a \mp \sqrt{b}}$.

To show a statement is an identity, expand and simplify the expressions on one or both sides of the equals sign, until the two expressions are the same.

A function is a rule for working out values of y when given values of x e.g. $y = 3x$ and $y = x^2$

The notation $f(x)$ is read as 'f of x'.

fg is the composition of the function f with the function g. To work out $fg(x)$, first work out $g(x)$ and then substitute your answer into $f(x)$.

The inverse function reverses the effect of the original function. $f^{-1}(x)$ is the inverse of $f(x)$.

To prove a statement is not true you can find a **counter-example** – an example that does not fit the statement.

For an algebraic proof let *n* represent any integer

Even number	$2n$
Odd number	$2n + 1$ or $2n - 1$
Consecutive numbers	$n, n + 1, n + 2, \dots$
Consecutive even numbers	$2n, 2n + 2, 2n + 4, \dots$
Consecutive odd numbers	$2n + 1, 2n + 3, 2n + 5, \dots$

Unit 18 Higher Vectors and Proof V353 V353a

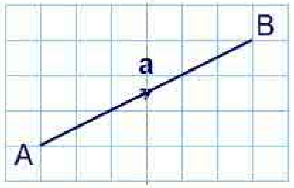
A **vector** is a quantity that has both size (or magnitude) and direction.

Examples of vector quantities are:
- displacement
- velocity
- force

A **scalar** is a quantity that has size (or magnitude) only.

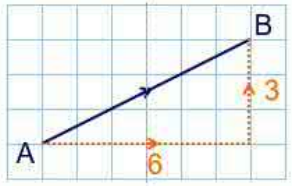
Examples of scalar quantities are:
- length
- speed

Vectors are written as **bold** lower case letters: **a**, **b**, **c**. When handwriting, underline the letter: a, b, c .



This vector goes from the point A to the point B.

We can write this vector as $\overrightarrow{AB}$.



To go from the point A to the point B we must move 6 units to the right and 3 units up.

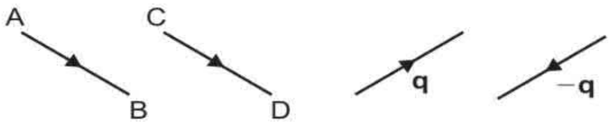
We can represent this movement using a **column vector**.

$\overrightarrow{AB} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

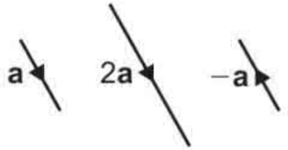
This is the horizontal component. It tells us the number of units in the x-direction.

This is the vertical component. It tells us the number of units in the y-direction.

If $\overrightarrow{AB} = \overrightarrow{CD}$ then the line segments AB and CD are equal in length and are parallel. $\overrightarrow{AB} = -\overrightarrow{BA}$



2**a** is twice as long as **a** and in the same direction.
−**a** is the same length as **a** but in the opposite direction.



With the origin O, the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are called the **position vectors** of the points A and B. In general, a point with coordinates (p, q) has position vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

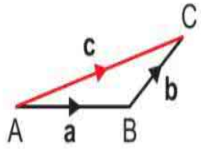
In general, if the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is multiplied by the scalar k, then

$$k \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

For example, $3 \times \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ 15 \end{pmatrix}$

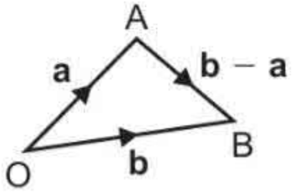
When a vector is multiplied by a scalar the resulting vector is either parallel to the original vector or lies on the same line.

Triangle law for vector addition: Let $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$.
Then $\mathbf{a} + \mathbf{b} = \mathbf{c}$ forms a triangle.

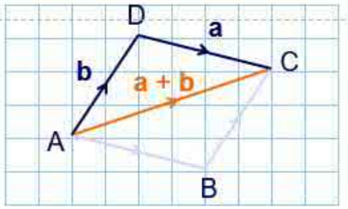


When $\mathbf{c} = \mathbf{a} + \mathbf{b}$ the vector **c** is called the **resultant vector** of the two vectors **a** and **b**.

When $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \mathbf{b} - \mathbf{a}$.



Suppose $\mathbf{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$



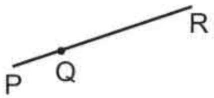
From this diagram we can see that

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + \mathbf{b}$

Also

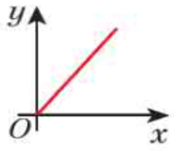
$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = \mathbf{b} + \mathbf{a}$

$\overrightarrow{PQ} = k\overrightarrow{QR}$ shows that the lines PQ and QR are parallel. Also they both pass through point Q so PQ and QR are part of the same straight line. P, Q and R are said to be **collinear** (they all lie on the same straight line).



Unit 19 Higher Proportion and Graphs V345 V255 V254

When a graph of two quantities is a straight line through the origin, one quantity is directly proportional to the other.



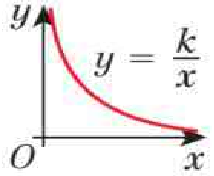
The symbol $\propto$ means 'is directly proportional to'.

If y is directly proportional to x, $y \propto x$ and $y = kx$, where k is a number, called the **constant of proportionality**.

Where k is the constant of proportionality:

- if y is proportional to the square of x then $y \propto x^2$ and $y = kx^2$
- if y is proportional to the cube of x then $y \propto x^3$ and $y = kx^3$
- if y is proportional to the square root of x then $y \propto \sqrt{x}$ and $y = k\sqrt{x}$

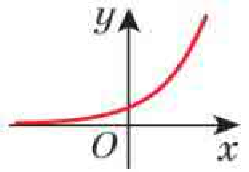
When y is **inversely proportional** to x, $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$



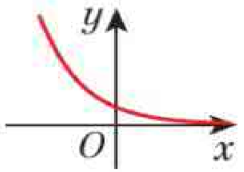
The tangent to a curved graph is a straight line that touches the graph at a point. The gradient at a point on a curve is the gradient of the tangent at that point.

Expressions of the form a^x or a^{-x}, where $a > 1$, are called **exponential functions**.

The graph of an exponential function has one of these shapes.



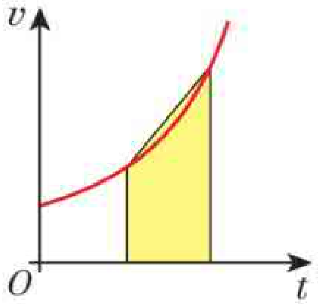
$y = a^x$ where $a > 1$ or
$y = b^{-x}$ where $0 < b < 1$
exponential growth



$y = a^{-x}$ where $a > 1$ or
$y = b^x$ where $0 < b < 1$
exponential decay

Exponential graphs intersect the y-axis at (0, 1) because $a^0 = 1$ for all values of a.

The area under a velocity–time graph shows the displacement, or distance from the starting point. To estimate the area under a part of a curved graph, draw a chord between the two points you are interested in, and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an estimate for the area under this part of the graph.



The gradient of the chord gives the average rate of change

Exmouth Community College

Higher: Transformation of Graphs Corbett Maths link:[Transformations-of-graphs]

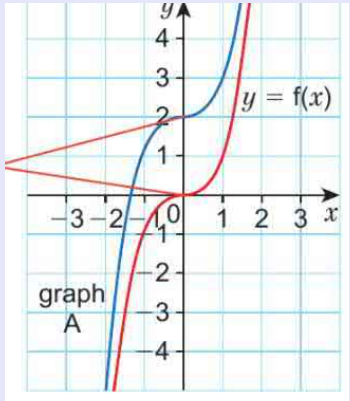
The graph of $y = \mathrm{f}(x)$ is transformed into the graph of:

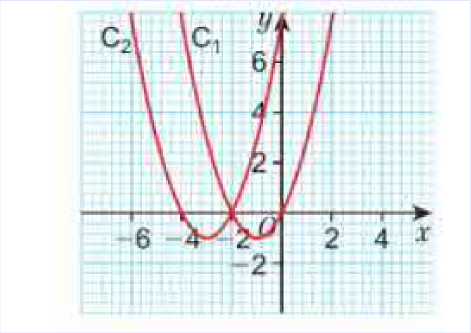
$y = \mathrm{f}(x) + a$ by a translation of a units parallel to the y-axis or a translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$

The graph of $y = \mathrm{f}(x)$ is transformed into the graph of:

$y = \mathrm{f}(x) + a$ by a translation of a units parallel to the y-axis or a translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$

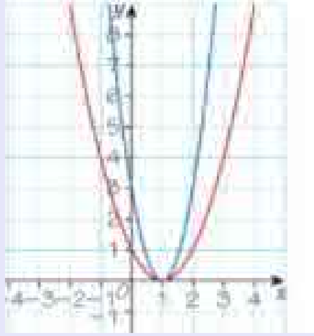
$y = \mathrm{f}(x + a)$ by a translation of $-a$ units parallel to the x-axis or a translation by $\begin{pmatrix} -a \\ 0 \end{pmatrix}$





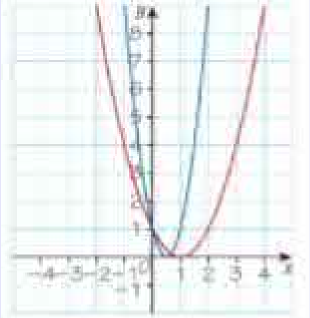
$y = \mathrm{f}(-x)$ by a reflection in the y-axis

$y = -\mathrm{f}(x)$ by a reflection in the x-axis



$y = a\mathrm{f}(x)$ by a stretch of scale factor a parallel to the y-axis

$y = \mathrm{f}(ax)$ by a stretch of scale factor $\frac{1}{a}$ parallel to the x-axis



edexcel

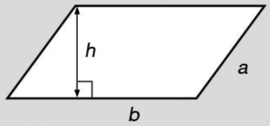
Edexcel GCSE (9-1) Maths: need-to-know formulae

www.edexcel.com/gcsemathsformulae

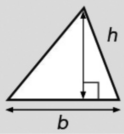
Areas

Rectangle = $l \times w$

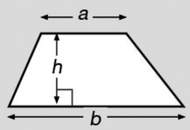
Parallelogram = $b \times h$



Triangle = $\frac{1}{2} b \times h$

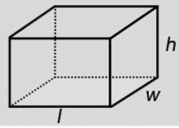


Trapezium = $\frac{1}{2}(a + b)h$

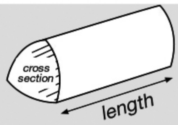


Volumes

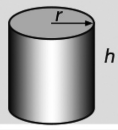
Cuboid = $l \times w \times h$



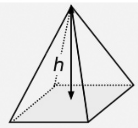
Prism = area of cross section $\times$ length



Cylinder = $\pi r^2 h$



Pyramid = $\frac{1}{3} \times$ area of base $\times h$

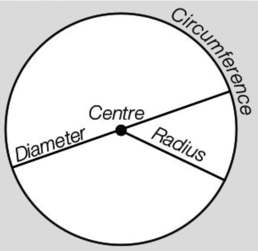


Circles

Circumference = $\pi \times$ diameter, $C = \pi d$

Circumference = $2 \times \pi \times$ radius, $C = 2\pi r$

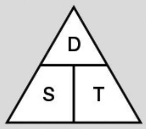
Area of a circle = π x radius squared, $A = \pi r^2$



Compound measures

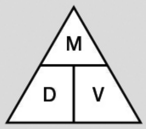
Speed

speed = $\frac{\text{distance}}{\text{time}}$



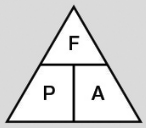
Density

density = $\frac{\text{mass}}{\text{volume}}$



Pressure

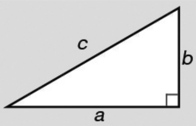
pressure = $\frac{\text{force}}{\text{area}}$



Pythagoras

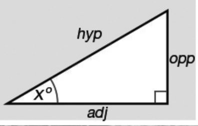
Pythagoras' Theorem

For a right-angled triangle, $a^2 + b^2 = c^2$



Trigonometric ratios (*new to F*)

$\sin x^\circ = \frac{\text{opp}}{\text{hyp}}$, $\cos x^\circ = \frac{\text{adj}}{\text{hyp}}$, $\tan x^\circ = \frac{\text{opp}}{\text{adj}}$

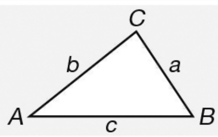


Trigonometric formulae

Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$



Quadratic equations

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Foundation tier formulae | Higher tier formulae

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