Expanding and factorising quadratics (double brackets)

Expanding a quadratic is just like multiplying 2-digit numbers – use a multiplication grid, then add your answers:

$$(x+2)(x+3) = x^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$

×	(x	+2)
(x	x^2	$+2x$
+3)	$+3x$	$+6$

(2+3) (2x3)

It's no coincidence!

Video 14: Expanding quadratics

Factorising a quadratic is the opposite of expanding it – you're putting it back into brackets (if you can). You can still use the grid, but do it in reverse:

$$x^2 + 7x + 12 = (x+3)(x+4)$$

×	(x	+3)
(x	x^2	$+3x$
+4)	$+4x$	$+12$

We know from expanding that the two numbers in my brackets will add to make 7, and multiply to make 12, so they must be 3 and 4 ($3x + 4x = 7x$ and $3 \times 4 = 12$)

Video 118: Factorising quadratics

Solving quadratics

Quadratic equations are written as equal to y, like so:

$$y = x^2 + bx + c$$

To find the solutions, we make them equal to 0 because the "*solutions*" are the "*x-intercepts*", where the graph crosses the x-axis. On the x-axis, the y-value would be zero (because we haven't moved up or down).

$$x^2 + 7x + 12 = 0$$

Then we can factorise to give two answers (one of the brackets must = 0).

$$(x+3)(x+4) = 0$$
$$x + 3 = 0 \text{ or } x + 4 = 0$$
$$x = -3 \text{ or } x = -4$$

Video 266: Solving quadratics by factorising

If we can't factorise (sometimes the numbers don't work), we can use the **quadratic formula**:

$$when\ x^2 + bx + c = 0, \qquad x = \frac{b^2 \pm \sqrt{4c}}{2}$$

Video 267: Using the quadratic formula

Plotting a Quadratic Graph

To plot a quadratic, make the expression equal to y, then make a table using different values of x. For example:

$$y = x^2 - 4x + 5$$

If $x = 1, y = (1)^2 - (4 \times 1) + 5$

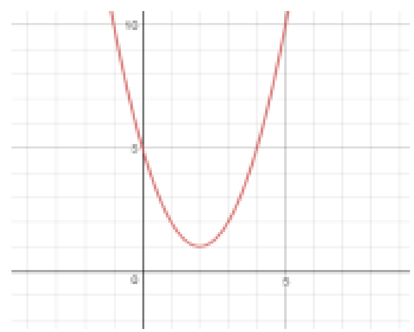
If $x = 1, y = 2$

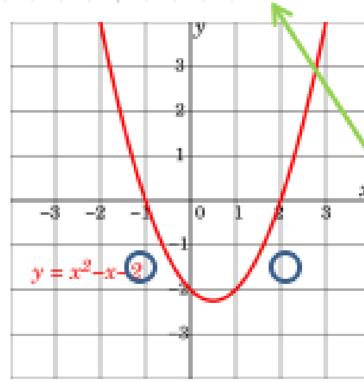
x	0	1	2	3	4
y	5	2	1	2	5

Video 264: Plotting a quadratic graph

Video 265: Sketching a quadratic graph using key coordinates

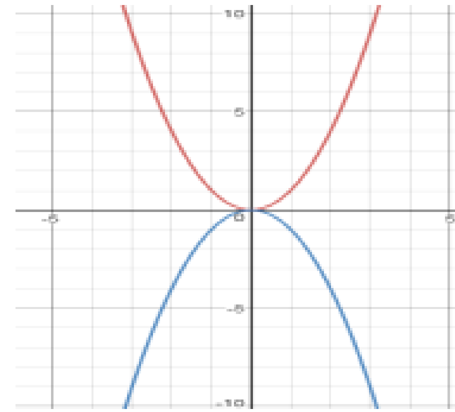
Based on the table above, the coordinates to plot would be: **(0, 5) (1, 2) (2, 1) (3, 2) (4, 5)**





Recognising a quadratic shape

All $y = x^2$ graphs will have the same symmetrical curved shape you see below, even if you can't see all of it. At any point on the line, the y-coordinate is the **square** of the x=coordinate



The upside down graph shows the equation $y = -x^2$, which is just the reflection of the positive version (the y-values have all become negative).

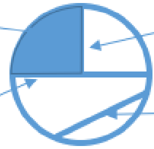
On the diagram, the solutions are -1 and 2 (circled), because that's where y = 0.

Some quadratics (like the one over there) do not cross the x-axis. This means they have no "solutions", because the y value never reaches 0!

Video 61

Key Facts

Circumference = perimeter of a circle (units)
Area = space inside a 2D shape (units^2)
Volume = the space inside a 3D shape (units^3)

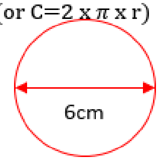


Circumference = video 60 Area = Video 40

Circumference and Area of Circles

Circumference $= \pi \times$ *diameter* (or $C = 2 \times \pi \times r$)

Area $= \pi \times$ *radius*^{2}

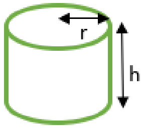


"πr^2 sounds like area to me, if you need the circumference you just use πd"

Remember our radius is half of our diameter

Volume and SA of cylinders

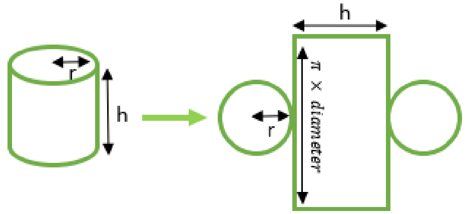
Volume = $\pi r^2 h$
V = $\pi \times radius^2 \times height$



(this is just the area of one of the circles multiplied by how long your cylinder is)

Volume = video 357

Surface Area



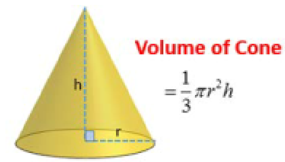
SA = 2 circle areas + rectangle area
SA = $2\pi r^2 + \pi dh$

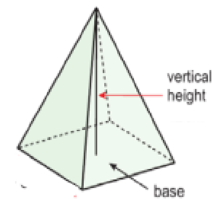
SA = video 315

Volume of pyramids and cones

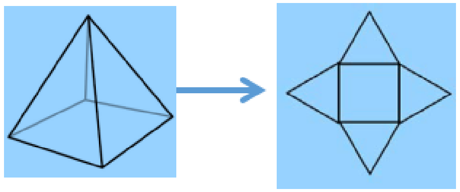
Volume of a Pyramid/Cone $= \frac{1}{3} \times$ *area of base* $\times$ *vertical height*

Volume of cone = video 359 Volume of pyramid = video 360



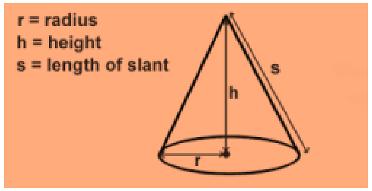


Surface Area of a Pyramid = total area of all faces



Area of all 4 triangles + area of the base

Surface Area of a Cone $= \pi \times radius \times$ *slant height* $= \pi r l$



SA of Cone = video 314

Arc length = video 58

Semicircles and Sectors

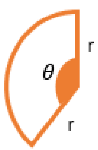
Perimeter of semi-circle = video 62

Perimeter

Perimeter = arc length + radius + radius

Arc length $= \frac{\theta}{360} \times \pi \times diameter$

Arc length is a fraction of the circumference



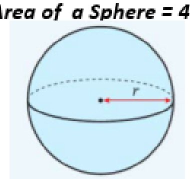
Area Video 46

Area $= \frac{\theta}{360} \times \pi \times radius^2$

Volume and surface area of spheres

Volume of Sphere $= \frac{4}{3}\pi r^3$

Surface Area of a Sphere $= 4\pi r^2$



SA = video 313 Volume = video 361

Multiplying and dividing fractions

To multiply fractions, just multiply the ***numerators*** and multiply the ***denominators*** (then simplify if you can!)

$$\frac{2}{3} \times \frac{3}{5} = \frac{2\times3}{3\times5} = \frac{6}{15} \ (= \frac{2}{5})$$

Multiplying fractions

To divide by a fraction, multiply by the ***reciprocal*** (flip the numerator and denominator)

Dividing fractions

$$\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{2\times5}{3\times3} = \frac{10}{9}$$

Combining indices

When multiplying indices with the same base value, ***add* the powers**:

$2^2 \times 2^3 = \underline{2 \times 2} \times \underline{2 \times 2 \times 2}$, so

$2^2 \times 2^3 = 2^{(2+3)} = 2^5$

Video 174: Laws of indices (including power of 0)

When dividing indices with the same base value, ***subtract* the powers**:

$$3^6 \div 3^2 = \frac{3\times3\times3\times3\times3\times3}{3\times3}$$

$$3^6 \div 3^2 = 3^{(6-2)} = 3^4$$

Power of 0

Anything to the power of 0 is equal to 1, no matter what it is! We can show this by dividing two identical indices:

$3^2 \div 3^2 = 3^{(2-2)}$

$3^2 \div 3^2 = 3^0$

Since dividing a value by itself always gives the answer 1, we also know that:

$3^2 \div 3^2 = 1$, therefore $\mathbf{3^0 = 1}$

This works for all numbers AND letters!

Reciprocals

When two numbers are reciprocal, it means they ***multiply to make 1*** (they're a bit like "opposites").

So 2 and ½ are reciprocal because

2 x ½ = 1

Reciprocal fractions are the ***reverse*** of each other, as shown:

$$\frac{2}{3} \times \frac{3}{2} = \frac{2\times3}{3\times2} = \frac{6}{6} = 1$$

The numerator will always match the denominator, and we know that anything divided by itself is 1!

Negative indices

Raising something to the power is the same as raising the ***reciprocal*** (see left) to the positive power.

Video 175: Negative indices

$$3^{-2} = \left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} = \frac{1}{3^2}$$

It's no coincidence!

$$\text{Negative power} = \frac{1}{positive\ power}$$

Standard form

Standard form is a way of writing very large or very small numbers using powers of 10 (multiplying/dividing by 10 until the decimal point is in the right place). The base number must always be between 1 and 10.

Video 300: Standard form

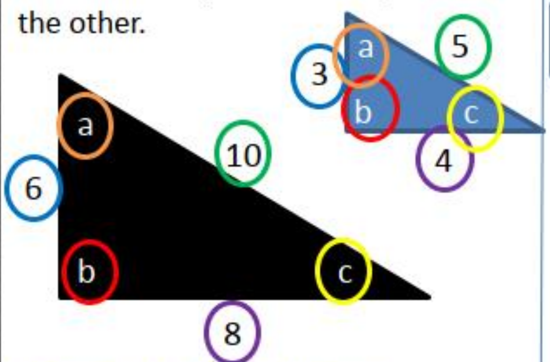
e.g. 5000000000000000000
can be written as
5 x 1000000000000000000,
which can then be written as
$\mathbf{5 \times 10^{18}}$
Clearly, the last way is quicker!

$0.00000000004 = 4 \div 100000000000$
$= 4 \times 10^{-11}$

NOTE:
When we divide, we use negative powers!

SIMILARITY

When shapes look the same but are different sizes, they are mathematically ***similar***. This means their *corresponding* ("matching") **angles** are **equal**, and their *corresponding* **sides** are in the **same ratio**. One shape is an *enlargement* of the other.



Congruence & Similarity definitions

How to find missing sides

VECTORS

Column vectors describe horizontal and vertical "movement", a bit like how co-ordinates describe position. They look similar, but they're arranged in a column (hence the name), as shown below:

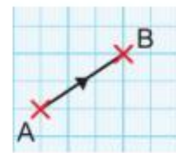
Column vectors

$\begin{pmatrix} x \\ y \end{pmatrix}$ x horizontal movement, y vertical movement

To get from A to B, you go 3 right, 2 up:

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ (**a**)

Reverse: $\overrightarrow{BA} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ (-**a**)



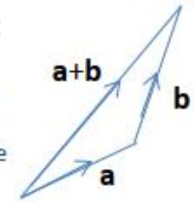
Vectors are labelled with a lower case letter, either **bold** or underlined.

You can combine vectors by adding their x and y values to give a ***resultant*** vector:

$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\mathbf{a}+\mathbf{b} = \begin{pmatrix} 3+4 \\ 2+1 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \end{pmatrix}$

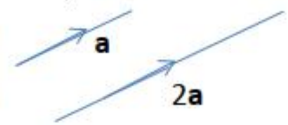
It would look like this:

We do this to move between points that don't have a vector between them – you can only go the way you know!

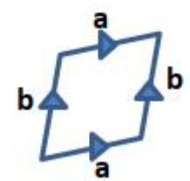


Vectors can also be multiplied:

$2\mathbf{a} = \begin{pmatrix} 3\text{x}2 \\ 2\text{x}2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$



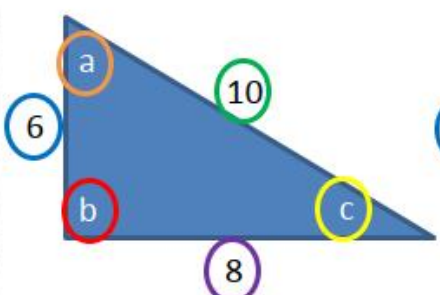
Parallel vectors can be represented using the same letter:

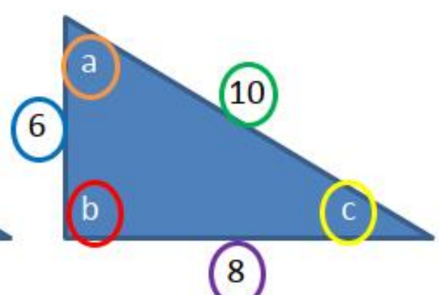


Algebraic vectors

CONGRUENCE

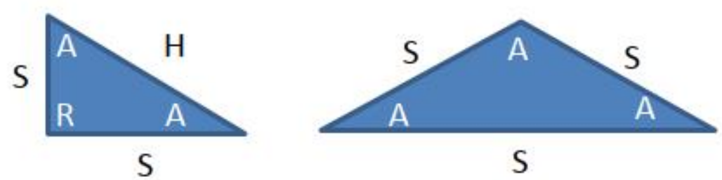
When shapes are identical, they are ***congruent***. All *corresponding* lengths and angles are **equal** – you could fit one perfectly on top of the other.





You can prove two triangles are congruent by showing that any of these combinations are matching (video here):

SSS (all three sides)
SAS (two sides and the angle between them)
ASA (two angles and the side which connects them)
AAS (two angles and the side after the second angle)
RHS (right angle, hypotenuse and one other side)*



*only applies to right-angled triangles

Year 11 Mathematics – Unit 20 – More Algebra – Autumn Term

Quadratic functions contain a term in x^2 but no higher power of x.
Video 266 - https://tinyurl.com/y8san5jm

Cubic functions contain a term in x^3 but no higher power of x.
Video 344 - https://tinyurl.com/yamclpto

Cubic functions can contain terms in x^2, x, and number terms.

When a cubic function is equal to zero it may have one, two, or three solutions. The solution to a cubic function equalling zero is there the graph crosses the x-axis. The solutions are commonly called **roots**.
Video 264 - https://tinyurl.com/y7u3d79a

The **reciprocal** function ($y = \frac{1}{x}$) of a cubic function has the x- and y-axes as **asymptotes** to the graph.
Video 346 - https://tinyurl.com/yd8x2uz8

An **asymptote** is a line that the graph gets closer and closer to, but never actually touches.

When a graph has x and y in **direct proportion**, $y = kx$
Video 254 - https://tinyurl.com/htma465

When a graph has x and y **inversely proportional** to each other, $y = \frac{k}{x}$
Video 255 - https://tinyurl.com/yb2ur2yq

The graph of two quantities that are inversely proportional is a reciprocal graph.

Simultaneous equations are equations that are both true for a pair of variables (letters).

Video 296 - https://tinyurl.com/y9dbmoee

Simultaneous equations can be solved graphically by plotting both equations on the same coordinate grid. The point at which the lines cross (the point of **intersection**) has the coordinates that are the solution.

Simultaneous equations can also be solved by the elimination method. To do this, the coefficients of either the x or y terms must be equal (or equal with the opposite sign).
Video 295 - https://tinyurl.com/yadevfgk
Subtract (or add) the two equations to eliminate one of the terms. The remaining term can now be evaluated.

The **subject** of a formula is the letter on its own side of the equals sign.
Video 7 - https://tinyurl.com/yc6vax5f
You can change the subject of a formula using **inverse operations** (subtract to move an added term to the other side, etc).
Video 8 - https://tinyurl.com/yahmeoyn
An **even number** is a multiple of 2. $2m$ and $2n$ are general terms for even numbers where m and n are integers.

An **equation** has an equals sign (=). You can solve it to find one value of the letter (unknown/variable).

An **identity** has an equivalent (triple bar) sign (≡). The left hand side equals the right hand side for all values of the letter (unknown/variable).

Key Points:

https://tinyurl.com/ybfxnjsj

Knowledge Check:

https://tinyurl.com/y9nl3tka