YEAR 9 – REASONING WITH ALGEBRA...

Straight Line Graphs

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use y= mx + c
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.

Parallel: two lines that never meet with the same gradient.

Co-ordinate: a set of values that show an exact position on a graph.

Linear: linear graphs (straight line) – linear common difference by addition/ subtraction

Asymptote: a straight line that a graph will never meet.

Reciprocal: a pair of numbers that multiply together to give 1.

Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes

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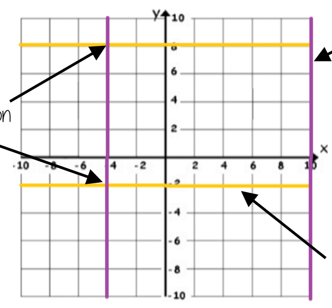


Intersection points

All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

Plotting y = mx + c graphs

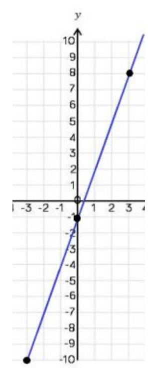
R

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

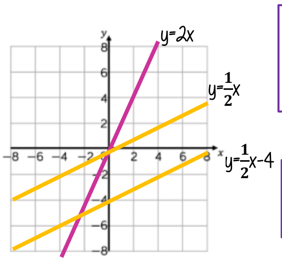
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



The **greater** the gradient – the **steeper** the line

Parallel lines have the **same** gradient

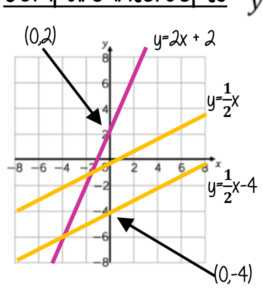
Positive gradients

Negative gradients

Compare Intercepts

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

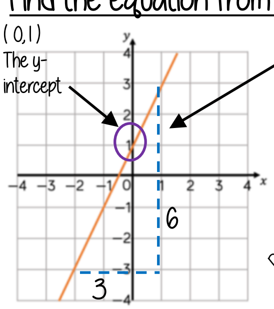
The equation of a line can be rearranged: Eg:

y = c + mx

c = y – mx

Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin.

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

When you have 0 pens this has 0 cost. The gradient shows the price per pen

YEAR 9 – REASONING WITH ALGEBRA...

Forming and Solving Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets

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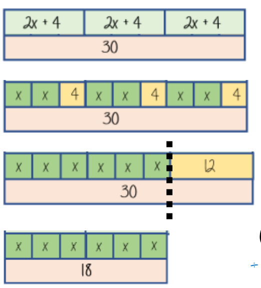


$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

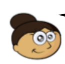
$6x = 18$

$\div 6 \quad \div 6$

$\underline{x = 3}$

Form and solve inequalities

R



Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

$x \leftarrow -3 \leftarrow -2 \leftarrow 11$

$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$

$+3x \quad +3x$

$2 > 17 + 3x$

$-17 \quad -17$

$-15 > 3x$

$\div 3 \quad \div 3$

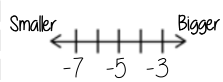
$\underline{-5 > x}$

x is true for any value smaller than -5

CHECK IT!

$2 - 3(-6) = 20$

TRUE/ CORRECT



Method 2 Keep the negative x

$2 - 3x > 17$

$-2 \quad -2$

$-3x > 15$

$\div -3 \quad \div -3$

$\underline{x > -5}$

x is true for any value bigger than -5

This cannot be true...

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

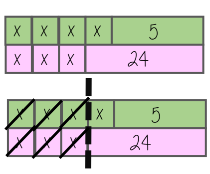
$4x + 5 = 3x + 24$

$-3x \quad -3x$

$x + 5 = 24$

$-5 \quad -5$

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$

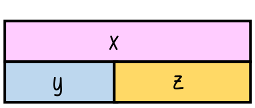
✓ -20 IS smaller than -18

Formulae and Equations

Substitute in values

Formulae – all expressed in symbols → **Equations** – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the subject.

$\underline{y = x - z}$

$y \rightarrow +z \rightarrow x$

$y \leftarrow -z \leftarrow x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging...

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$

$+3 \quad +3$

$4x = 12$

$\div 4 \quad \div 4$

$\underline{x = 3}$

In a formula (make x the subject)

$xy - s = a$

$+s \quad +s$

$xy = a + s$

$\div y \quad \div y$

$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \dfrac{4x + 9}{2}$

Gradient $= \dfrac{4}{2} = 2$

YEAR 9 – REASONING WITH ALGEBRA...

Testing conjectures

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

Keywords

Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Prime: an integer with only 2 factors.
HCF: highest common factor (biggest factor two or more numbers share)
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Verify: the process of making sure a solution is correct
Proof: logical mathematical arguments used to show the truth of a statement
Binomial: a polynomial with two terms
Quadratic: a polynomial with four terms (often simplified to three terms)

Factors, Multiples and Primes

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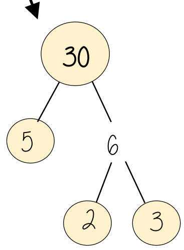
Multiplication part-whole models



All three prime factor trees represent the same decomposition

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, (6), 9, 18

30: 1, 2, 3, 5, (6), 10, 15, 30

Common factors are factors two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, (36), 45, 54

12: 12, 24, (36), 48, 60

Common multiples are multiples two or more numbers share

True or False?

Conjecture

A pattern that is noticed for many cases

1, 2, 4.....
The numbers in the sequence are doubling each time.

Counterexamples

This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

Always, Sometimes, Never true.

Always – Every value always supports the statement

Sometimes – Examples show the statement being true and counter examples to show when it is false.

Never – No example supports the statement

Examples to try
- 0 and 1
- Fractions
- Negative numbers

Show that

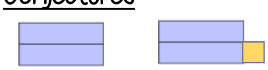
Numerical verification – Show the stages to a solution with numerical values

Algebraic verification – Show algebraic properties of the solution. You may want to use pictorial images to support this

Proof – Simple proofs using algebra

Compare the left hand side of an equation with the right hand side – are they the same or different?

Conjectures



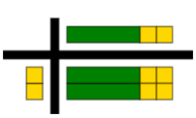
Even $(2n)$ Multiple of 2

Odd $(2n+1)$ One more than any even

Use numerical verification first
Use pictorial verification – the representations of numbers of odd and even

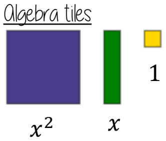
Expanding binomials

$2(x+2) \equiv 2x+4$



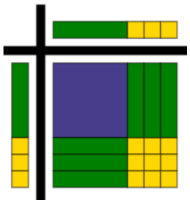
Algebra tiles can represent a binomial expansion
Has two terms

Algebra tiles



Positive values

$(x+3)(x+3) \equiv x^2+6x+9$



This is a quadratic. It has four terms which simplified to three terms

The order of the binomial has no impact on the outcome. eg $(x+3)(3+x)$

Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers

Positions of numbers in relation to n form expressions.
Eg one space to the right of n
$n+1$

Eg One row below n
$n+10$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The size of the grid for generalisation changes the relationship statements

YEAR 9 – CONSTRUCTING IN 2D/3D...

3D Shapes

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

2D: two dimensions to the shape e.g. length and width

3D: three dimensions to the shape e.g. length, width and height

Vertex: a point where two or more line segments meet

Edge a line on the boundary joining two vertex

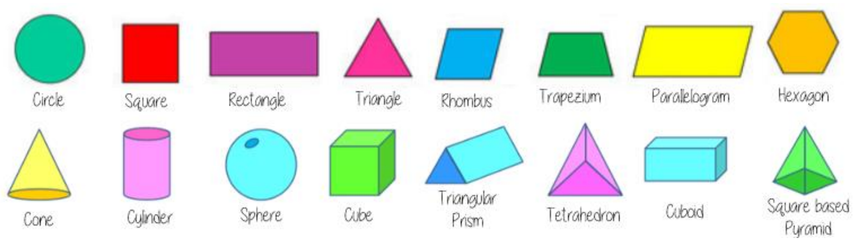
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

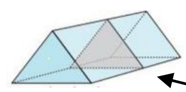
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

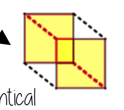
Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides

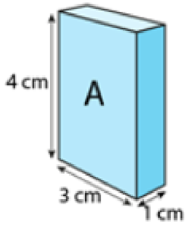


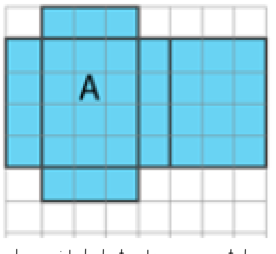


The cross section will also be identical to the end faces.

A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

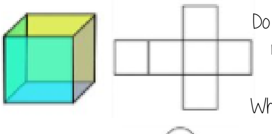




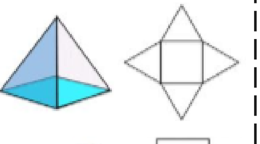
1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

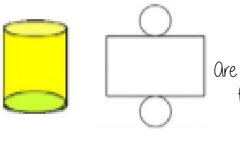
Sketch and recognise nets



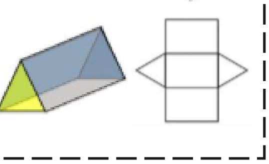
Do they have the same number of faces?



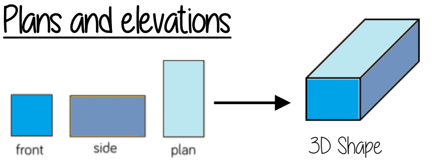
Where do the edges join?



Are the shapes of the faces correct?



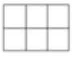
Plans and elevations



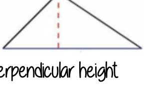
The direction you are considering the shape from determines the front and side views

Area of 2D shapes

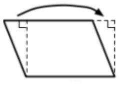
Rectangle
Base x Height



Triangle
½ x Base x Perpendicular height

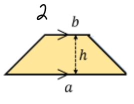


Parallelogram/ Rhombus
Base x Perpendicular height



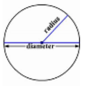
Area of a trapezium

$\frac{(a+b) \times h}{2}$



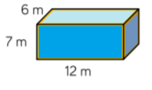
Area of a circle

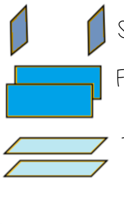
π x radius²



Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area

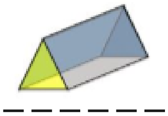




Sides	6 x 7	
	6 x 7	
Front and back	12 x 7	
	12 x 7	
Top and Bottom	12 x 6	
	12 x 6	

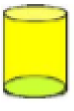
Sum of all sides is surface area

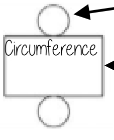
For cubes and cuboids you can also find one of each face and double it



For other shapes – not all the sides are the same, so calculate the individually

Surface area - cylinders





The area of the circle
π x radius²

The width of this face is the same as the circumference
π x diameter x height

$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$

Volumes

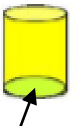
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

Counting cubes

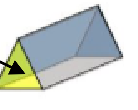
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



Cross section

Prisms and cylinders = area cross section x height

Height can also be described as depth

Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

YEAR 9 – CONSTRUCTING IN 2D/3D...

Constructions & congruency

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

Keywords

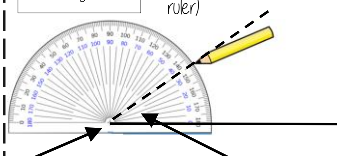
Protractor: piece of equipment used to measure and draw angles
Locus: set of points with a common property
Equidistant: the same distance
Discorectangle: (a stadium) – a rectangle with semi circles at either end
Perpendicular: lines that meet at 90°
Arc: part of a curve
Bisector: a line that divides something into two equal parts
Congruent: the same shape and size

Draw and measure angles

R

Draw a 35° angle

Make a mark at 35° with a pencil
And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

R

A picture of a car is drawn with a scale of 1:30

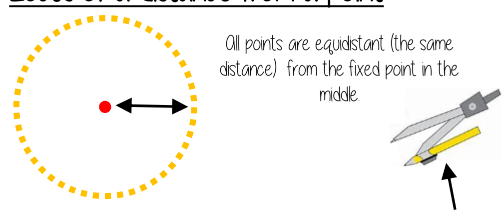
For every 1cm on my image is 30cm in real life

The car image is 10cm

Image	Real life
1cm	30cm
10cm	300cm

× 10   × 10

Locus of a distance from a point

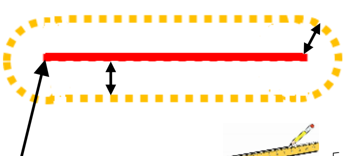


All points are equidistant (the same distance) from the fixed point in the middle.

Equipment needed
The radius is the distance from the fixed point

If the point is in the corner it can only make a quarter circle

Locus of a distance from a straight line

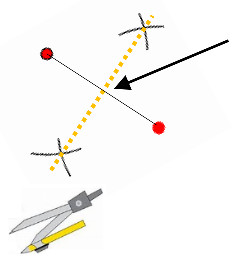


All points are equidistant (the same distance) from line

The ends of the line are fixed points

Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points

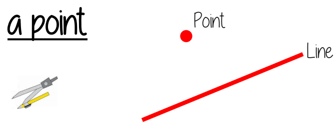


Also a perpendicular bisector
Because if the points are joined, this new line intersects it at a 90°

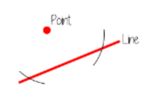
Keep the compass the same size and draw two arcs from each point

Join the intersections with a ruler.
All points on this line are equidistant from both points

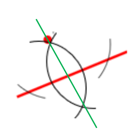
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



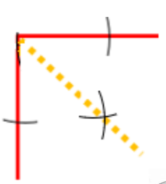
Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



Also an angle bisector
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

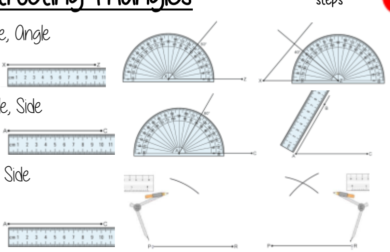
Constructing Triangles

Link to steps → R

Side, Angle, Angle

Side, Angle, Side

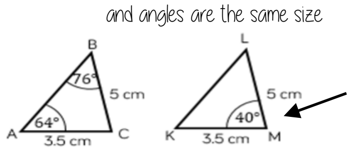
Side, Side, Side



Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

Congruent triangles

Side-side-side
All three sides on the triangle are the same size

Angle-side-angle
Two angles and the side connecting them are equal in two triangles

Side-angle-side
Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side
The triangles both have a right angle, the hypotenuse and one side are the same